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KEY TO MODEL ALGEBRA

FOR

ELEMENTARY SCHOOLS.

1. 18 apples. **2.** 8. **3.** 5 books, Mary; 20 books, Anna. **4.** 12 dollars. **5.** 12 dollars, B; 84 dollars, A. **6.** 12. **7.** 4 fish, Henry; 32 fish, William. **8.** 24. **9.** \$40, carriage; \$120, horse. **10.** 9. **11.** 17 miles. **12.** 2 years, Ida; 16 years, Emma. **13.** 12 marbles. **14.** 20 pigeons. **15.** 20 rabbits, Frank; 4 rabbits, George. **16.** 12 oranges, Martha; 3 oranges, Anna. **17.** 18 years, John; 12 years, Mary. **18.** 50 cents. **19.** 25 fish, James; 10 fish, Thomas. **20.** 36 years, B; 27 years, A. **21.** \$80, horse; \$60, carriage. **22.** 36, 8. **23.** 24 quarts, George; 9 quarts, Anna. **24.** \$6, sheep; \$21, cow. **25.** 8 cents, first; 16 cents, second; 32 cents, third. **26.** 6, first; 24, second; 30, third. **27.** 14 years, Ida; 8 years, Emma. **28.** 80 sheep, Mr. Brown; 50 sheep, Mr. Jones. **29.** 4 oranges, Minnie; 12 oranges, Harry; 8 oranges, Frank. **30.** 40 marbles. **31.** 12 oranges. **32.** 80 cents.

33. Let $x =$ the number,

Then $\underline{x+(\frac{1}{4}x-\frac{1}{5}x)=42.}$

$$x+\frac{1}{20}x=42$$

$$\frac{21}{20}x=42$$

$$\frac{1}{20}x=2$$

$$\frac{20}{20}x \text{ or } x=40$$

Therefore, the number is 40.

34. 15 years. **35.** 60 cents. **36.** 16 feet, longer; 11 feet, shorter. **37.** 18. **38.** \$40, cart; \$80, pony. **39.** 4, 10. **40.** 45 pigeons. **41.** 18. **42.** 80 cents. **43.** \$40. **44.** \$10.

45. Let $x =$ Kate's age,

Then $\underline{x+12=20 \text{ years.}}$

$$x=20-12$$

$$x=8$$

Therefore, Kate is 8 years old.

(See explanation, Illustrative Problem 29.)

46. 12 marbles. **47.** 10 rabbits, Harry; 15 rabbits, Frank. **48.** \$50, carriage; \$70, horse. **49.** 20 cents, Alice; 30 cents, Lucy. **50.** 17 miles.

51. Let $x =$ father's age,
And $\frac{1}{4}x =$ Henry's age,
Then $\underline{x + \frac{1}{4}x = 60 \text{ yrs.} - 10 \text{ yrs.}}$
$\frac{5}{4}x = 50$
$\frac{1}{4}x = 10$
$\frac{4}{4}x$ or $x = 40$
Therefore, Henry is 10 years old, and his father is 40 years old.

52. 11. **53.** 5 quarts, William; 13 quarts, John. **54.** 30 feet.

55. Let $x =$ Walter's number of quarts,
And $x - 6 =$ Edgar's number,
Then $\underline{x + x - 6 = 30 \text{ quarts.}}$
$x + x = 30 + 6$
$2x = 36$
$x = 18$
$x - 6 = 12$
Therefore, Walter sold 18 quarts, and Edgar sold 12 quarts.
(See explanation, Illustrative Problem 30.)

56. 10. **57.** 14 oranges, Ida; 10 oranges, Emma. **58.** 12 rabbits, Frank; 8 rabbits, Emma. **59.** \$100. **60.** 54 cents. **61.** 14 years. **62.** \$60, watch; \$15, chain. **63.** 15 sheep. **64.** 12. **65.** 15 years. **66.** 16 marbles. **67.** 12 cents, lemons; 24 cents, oranges.

68. Let $x =$ Anna's,
And $\frac{2}{3}x =$ Martha's,
And $\frac{1}{2}x - 10 =$ Nellie's,
Then $\underline{x + \frac{2}{3}x + \frac{1}{2}x - 10 = 42}$ cents.
$\frac{13}{6}x = 52$
$\frac{1}{6}x = 4$
$\frac{6}{6}x$ or $x = 24$
$\frac{2}{3}x = 16$
$\frac{1}{2}x - 10 = 2$
Therefore, Anna has 24 cents, Martha, 16 cents, and Nellie, 2 cents.

69. 12, 2; 24.

70. Let $x =$ number John bought,
And $\frac{1}{2}x =$ number Thomas bought,
Then $\underline{2x + \frac{3}{2}x = 77}$ cents.
$\frac{7}{2}x = 77$

$\frac{1}{2}x = 11$
$\frac{2}{2}x$ or $x = 22$

Therefore, John bought 22 apples, and Thomas, 11 apples.

71. 15 oranges. **72.** 15. **73.** 360 yards. **74.** 24 feet. **75.** 8 inches. **76.** 8 inches long, 6 inches wide.

77. Let $x =$ amount in bank,
Then $x =$ amount earned,
And 20% of $(x+x) = \frac{2}{5}x$;
Or, $\frac{2}{5}x + 25 =$ amount spent,
Hence, $\underline{x + x - \frac{2}{5}x - 25 = 55}$ cents.
$1\frac{3}{5}x = 80$
$\frac{1}{5}x = 10$
$\frac{5}{5}x$ or $x = 50$

Therefore, Harry had 50 cents in his bank.

78. 12. **79.** 8 pigeons, George; 12 pigeons, William. **80.** 12 years. **81.** $1.20. **82.** 24 cents, Frank; 16 cents, Fanny. **83.** 25 cents. **84.** $10, father; $5, Frank; $4, Harry. **85.** 25 years, mother; 5 years, child; 30 years, father. **86.** 32 girls, 24 boys.

87. Let $x =$ B's money,
Then $\frac{2}{3}x =$ A's money,
And $\underline{x + \frac{2}{3}x = 75}$ cents.
$\frac{5}{3}x = 75$
$\frac{1}{3}x = 15$
$\frac{3}{3}x$ or $x = 45$
$\frac{2}{3}x = 30$

Therefore, A has 30 cents, and B has 45 cents.

88. $15, Henry; $10, George. **89.** $18, A paid; $12, B paid. **90.** $48, cost; $2, gain. **91.** $25, cost; $20, selling price at 20% loss.

92. Let $x =$ principal,
Then $\frac{1}{20}x =$ interest,
And $\underline{x + \frac{1}{20}x = \$210}$
$\frac{21}{20}x = 210$
$\frac{1}{20}x = 10$
$\frac{20}{20}x$ or $x = 200$

Therefore, the principal is $200.

93. $50. **94.** 36 marbles, Horace; 24 marbles, Herbert. **95.** 30 papers, first boy; 20 papers, second boy.

96. Let $x =$ C's,
Then $\frac{2}{3}x =$ B's,
And $\frac{1}{3}x =$ A's.
$\frac{3}{3}x =$ C's,
$\frac{1}{3}x =$ A's.
Therefore, A's $= \frac{1}{3}$ of C's, or
$\frac{1}{3}$ of C's $=$ A's.

97. Let $x =$ number of dimes,
And $2x =$ number of nickels,
Then $10x =$ value of dimes,
And $(2x \times 5)$, or $10x =$ value of nickels.
Hence, $10x + 10x = 100.$
$20x = 100$
$x = 5$
$2x = 10$
Therefore, you would give 5 dimes and 10 nickels.

98. $\frac{1}{2}$ of Frank's age $=$ John's age. (See solution of Problem 96.) **99.** 40 cents, first boy; 20 cents, second boy; 80 cents, third boy. **100.** 6 feet, pole; 18 feet, line.

Algebraic Expression.—Page 27.

1. $2ab^2.$
2. $\frac{x^3}{4}.$
3. $2a+b.$
4. $c^3-4d^2.$
5. $a+3bc^3.$
6. $3bc-d^3.$
7. $4c^2d+\frac{b}{3}.$
8. $4c^3+x^2y.$
9. $\frac{d^2}{2}-2a^3b.$
10. $a+3b+c^3.$
11. $\frac{bc}{3}-x^3.$
12. $4b^2c+\frac{a^2}{4}.$
13. $\frac{b^3}{4}-5c^2x^3.$
14. $4x^3+\frac{x^2}{2}.$
15. $2a+b-3.$
16. $3b-\frac{2}{3}cd+x^3.$
17. $5s-\frac{y^2}{4}.$
18. $\frac{a^2x}{5}+3y^3.$
19. $4b^3c-\frac{y^2}{2}.$
20. $2a+3b+4.$
21. $3b^2+5-\frac{2}{a}.$
22. $c^3-4d^2+\frac{ax}{5}.$
23. $4b^2c^3-\frac{2a}{b}.$
24. $5a^3x^2+4-\frac{2}{3}y.$

Page 29.

1. $a+b.$ **2.** $b+c.$ **3.** a cents $+ c$ cents. **4.** b cents $+ d$ cents. **5.** $a-b.$ **6.** $b-c$; b cents $- c$ cents. **7.** a dollars $- c$ dollars. **8.** x marbles $- y$ marbles, or $x-y$ is the difference in number. **9.** $a+a$, or $2a$; $a+a+a$, or $3a$. **10.** $3b$; $5b$. **11.** $3c$; $4c$. **12.** $d+3d$, or $4d$. **13.** $3x-2x$, or x. **14.** $3y$ cents. **15.** $\frac{a}{3}.$ **16.** $\frac{c \text{ cents}}{5}.$ **17.** $\frac{x \text{ cents}}{12}.$ **18.** $4-a.$ **19.** $\frac{b}{3}.$ **20.** $\frac{c}{3}$; $\frac{2c}{3}.$

$a=4$, $b=5$, $c=10$.

21. $5a$; 20. **22.** $5a$, 20; $4a$, 16. **23.** $7b$, 35; $3b$, 15. **24.** $9bc$, 450.

$a=5$, $b=7$, $c=12$.

25. $2a$, 10; $4a$, 20; $3a$, 15. **26.** $4b$, 28; $5b$, 35. **27.** $7b$, 49.
28. $5c$, 60; $7c$, 84. **29.** $2ac$, 120.

$a=3$, $b=5$, $c=7$.

30. $2a$, 6; $3a^2$, 27. **31.** $5b$, 25. **32.** $3b$, 15; ab, 15; b^2c^3, 8575.
33. $8c$, 56; $15b$, 75. **34.** $12ac$, 252.

$a=6$, $b=8$, $c=10$.

35. $\frac{a}{2}$, 3. **36.** $12c \div 6 = 2c = 20$; $15a \div 5a = 3$. **37.** $\frac{2a}{3} = 4$; $\frac{16}{b} = 2$.
38. $12ac \div 3a = 4c = 40$.

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39. $a+b$, 8 cents. **40.** $b+c$, 14 cans. **41.** $a+b+c$, 18 miles.
42. $a+b$, 18 cents. **43.** $a+b+c$, 20 yd.

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44. $a-b$, 7 years. **45.** $b-c$, 5 cents. **46.** $a-b-c$, 6 tons. **47.** $b-c-d$, 10 gal. **48.** $c-d-e$, 3. **49.** ab cents, 45 cts. **50.** bc yards, 60 yd. **51.** ab cents $+\, cb$ cents, 60 cts. **52.** ab cents $+\, b$ cents, 60 cts. **53.** ab miles $-\, bc$ miles, 5 miles.

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54. $a \div b$, or $\frac{a}{b}$ boys, 4 boys. **55.** $\frac{a}{b}$, 5; $\frac{a}{c}$, 2. **56.** $(a \div b)c$, or $\frac{ac}{b}$, \$14. **57.** $a \div bc$, or $\frac{a}{bc}$, 2. **58.** $\frac{ab}{c}$ miles, 12 miles.

$a=4$, $b=8$, $c=12$.

59. $a+b$, 12 cents. **60.** $c+5$, 17 oranges. **61.** $c+b+c+b$, or $2c+2b$, 40 inches. **62.** $c-a$, 8 cents. **63.** $2b-3$, 13 inches. **64.** $c-a$, 8 feet. **65.** $3a \times c$, or $3ac$, 144. **66.** $b \times 10$, or $10b$, 80 cents. **67.** $4a+3c$, 52 cents. **68.** $\frac{12}{a}$, or $12 \div a$, 3 apples. **69.** $5b \div 4$, or $\frac{5b}{4}$, 10 pounds. **70.** $4c \div a$, or $\frac{4c}{a}$, 12 hats. **71.** $32 \div b$, or $\frac{32}{b}$, 4 cents.

72. $c+\frac{1}{4}c$, or $\frac{5}{4}c$, \$15. **73.** $5c\times\frac{1}{20}$, or $\frac{c}{4}$, \$3; $5c\times\frac{1}{10}$, or $\frac{c}{2}$, \$6; $5c\times\frac{7}{40}$, or $\frac{7c}{8}$, \$10.50.

$a=10$, $b=12$, $c=20$, $x=50$.

74. $\frac{a}{5}$, 2. **75.** $b\times 2$, or $2b$, 24. **76.** $\frac{c}{4}$, 5. **77.** $\frac{3}{5}c$, 12. **78.** $\frac{4}{3}b$, 16. **79.** $\frac{c+10}{6}$, \$5. **80.** $15b-15a$, 30 cents. **81.** $\frac{ab}{2}$, 60. **82.** $x\div a$, 5. **83.** $10c$ cents $\div$ 100 cents, \$2. **84.** $\frac{c}{4}$, $\frac{3c}{4}$; 5, 15. **85.** $c-4$, 16 yr.; $c+a$, 30 yr. **86.** $c+a-b$, 18 rabbits. **87.** $x-\frac{a}{2}-b$, \$33. **88.** $\frac{3c}{b}$, 5 yd. **89.** $\frac{c}{5}$; 4 yr., Elsie; $\frac{4c}{5}$; 16 yr., William. **90.** $(b\times c)+a$, or $bc+a$, 250. **91.** $\frac{x-2}{b}$, 4. **92.** $abc-500$, \$19. **93.** $\frac{c}{4}$, \$5. **94.** $bx-ax$, \$1. **95.** $\frac{c+10}{a}$, \$3. **96.** $\frac{c}{5}$, 4 years. **97.** $\frac{x}{5}$, $\frac{4x}{5}$; 10 marbles, Thomas; 40 marbles, Henry. **98.** $x-\frac{x}{2}-a$, 15 cents. **99.** $\frac{bc}{2}\div 6a$, 2 pounds. **100.** $2c+\frac{1}{5}c$, \$44.

Positive and Negative Quantities.—Page 40.

$a=6$, $b=5$, $c=4$, $x=3$, $y=2$.

1. . . . **2.** . . . **3.** 6. **4.** -3. **5.** . . . **6.** . . .
7. -12. **8.** 24. **9.** . . . **10.** 108. **11.** 1500. **12.** -9.
13. -720. **14.** 72. **15.** -36. **16.** -4860. **17.** $120\div 3=40$.
18. $-\frac{b^2y}{5}$; $-\frac{50}{5}$, -10.

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19. 10. **20.** 12. **21.** 14. **22.** 17. **23.** -13. **24.** -15.
25. -18. **26.** -22. **27.** 18. **28.** -25. **29.** 29. **30.** -34.

$a=10$, $b=8$, $c=5$, $x=3$, $y=2$.

31. 18. **32.** 13. **33.** 30. **34.** 14. **35.** -40. **36.** -17.
37. -22. **38.** -28. **39.** $+14$. **40.** -28. **41.** 15. **42.** -22.

$a=2$, $b=3$, $c=4$, $x=5$, $y=6$.

43. -8. **44.** $+5$. **45.** 5. **46.** $+2$. **47.** 5. **48.** 7. **49.** $+4$.

50. 4. **51.** -1. **52.** 4. **53.** 0. **54.** $+6$. **55.** 4. **56.** -7. **57.** 13.

$$a=5,\ b=4,\ c=3,\ x=2,\ y=1.$$

58. a, 5. **59.** $-b$, -4. **60.** $2c$, 6. **61.** $-3x$, -6. **62.** $a+b$, 9. **63.** $-b-c$, -7. **64.** $2x+3x$, 10. **65.** $-3x-4y$, -10. **66.** $3a-2a$, 5. **67.** $-5c+4x$, -7. **68.** $-2b-3b-4b$, -36. **69.** $-3a-4b+5c$, -16. **70.** $3b-c+5b$, 29.

Illustrations of Positive and Negative Quantities.—Page 43.

$$\textbf{1.}\quad \begin{array}{r} +10 \text{ dollars,} \\ \underline{-10 \text{ dollars,}} \\ 0 \text{ dollars.} \end{array}$$

2. $+5$ dollars. **3.** -5 dollars. **4.** 0 degrees, —temperature as at first. **5.** $+2$ degrees. **6.** -2 degrees. **7.** -10 cents Emma (the worse off by 10 cents). **8.** 2 miles. **9.** -2 miles. **10.** $+8$ miles. **11.** $+40$ cents (gain). **12.** $+3$ cents. **13.** $+15$ cents. **14.** $+12$ feet. **15.** $(a+b-10)$ dollars. **16.** $(50-x+y)$ feet.

Equations.—Page 45.

$$\textbf{1.}\quad \begin{array}{r} x+3x+5x=36 \\ \underline{4x+12+20}=36 \\ 9x=36 \\ x=4. \end{array}$$

In like manner, after finding the value of x in problems 1–8, substitute its value and prove the equation.

NOTE.—The equation formed by substituting the value of x should not be inserted until the solution is completed.

2. 4. **3.** 10. **4.** 12. **5.** 5. **6.** 3. **7.** 4. **8.** 15. **9.** 14 inches, 22 inches. **10.** 5 oranges, Frank; 8 oranges, Harry; 17 oranges, Walter. **11.** 60. **12.** 9 miles, first; 12 miles, second; 19 miles, third. **13.** 7 years. **14.** \$60. **15.** 20. **16.** 3 cents. **17.** \$8, Mary; \$7, Harry. **18.** 11 marbles, George; 55 marbles, James; 33 marbles, William. **19.** 5 horses, 25 cows, 75 sheep. **20.** \$60 each cow, \$120 each horse. **21.** 6 hours. **22.** 2 cents, lemon; 4 cents, orange. **23.** 20.

24. Let $x =$ sum each put in,
Then $x - \$400 =$ sum B had after losing,
And $x + \$700 =$ sum A had after gaining,
Hence, $\underline{x + \$700 = 2x - \$800.}$
$x - 2x = -800 - 700$
$-x = -1500$
or $x = 1500$
Therefore, each invested $1500.

25. $25, harness; $100, carriage; $200, horse. **26.** 36. **27.** 38 pigeons. **28.** 36 oranges, Martha; 24 oranges, Mary. **29.** $2.

30. Let $x =$ interest last year,
Then $x + \frac{1}{20}x =$ interest this year,
And $\underline{x + x + \frac{1}{20}x = \$164.}$

$$\frac{41x}{20} = 164$$

$$x = 80$$

$$x + \tfrac{1}{20}x = 84$$

Therefore, the interest the first year was $80, and the second year, $84.

31. 18 oranges, John; 18 oranges, Thomas. **32.** 72 inches.

Addition.—Page 51.

1. $7ab$. **2.** $-13ax$. **3.** $4bc$. **4.** $-4bx$. **5.** $7ad$. **6.** ay. **7.** $3ab$. **8.** $-4ax$. **9.** $-2bc$. **10.** $10ay$. **11.** $2xy$. **12.** $-2am$. **13.** $-3bd$. **14.** $-4ay$. **15.** $-5b^2x$. **16.** $-6cm^2$. **17.** $-by^3$. **18.** $5\frac{3}{4}a^3c$. **19.** $5a$. **20.** $-5x$. **21.** $20b$. **22.** $-23c$. **23.** x. **24.** $-x$. **25.** $-4b$. **26.** $-5ax$. **27.** 0. **28.** 0. **29.** $-10a-3cx+ay$. **30.** $-7b-5ac-3x$. **31.** $-9c+b-3xy$. **32.** $-2ax$. **33.** $6x-4y$. **34.** $-2a-4c$. **35.** $-y$. **36.** $2x+y$. **37.** $3abx$. **38.** $14(a+x)$. **39.** $3a^2x$. **40.** $22x+3y$. **41.** $6a-x$. **42.** $2b+11x$. **43.** $16ab+5cd+17a^2b$. **44.** $-ac$. **45.** $-13(a+z)$. **46.** $6ay+8y^2+abx$. **47.** $27a^2x^3y+28a^4xy^2-21a^3x^5y$. **48.** $34ax-abc$. **49.** $-9a^2b^3c^4+36a^3b^2c^4+14a^4b^2c^3$. **50.** $-2(a+b)+3(x+y)$. **51.** $-3(a-b^2+c^3)$. **52.** $2x+3y$. **53.** $-3a+4b$. **54.** $ax-2y$. **55.** $3bc-ax$. **56.** $(a+b)c$. **57.** $(a-b)y$. **58.** 0. **59.** $2(a+b)$. **60.** $5(x+1)$. **61.** $(c-2)(x+y)$. **62.** $3a-2b-2a^2b$. **63.** $ab-5bc$. **64.** $3bc-3ax$. **65.** $3ax+3cd$. **66.** $(a-1)(a-x)$. **67.** $(2b+c)(x+y)$.

Algebraic Expression.—Page 53.

$a=8,\ b=6,\ c=4,\ x=50,\ y=10.$

1. $a+b$, 14. **2.** $4c+b$, 22. **3.** $b+x^2$, 2506. **4.** $a+bc$, 32. **5.** b^2c+y, 154. **6.** $ay+b^3$, 296. **7.** $4a+x-5a$, 42. **8.** $\frac{b+c+x}{y}$, 6. **9.** a^3-c^2, 496. **10.** $4bc-a^2$, 32. **11.** x cents $+\,y$ cents, 60 cents. **12.** $a+3$, 11 yr.; $a+b$, 14 yr. **13.** $2x+2y$, 120 rd. **14.** $x+a$, \$58. **15.** $6y+\frac{6}{10}y$, \$66. **16.** $x+\frac{1}{20}x$, \$52.50. **17.** $a+1$, 9; $a-2$, 6. **18.** $a+b+c$, 18 yr. **19.** $\frac{ab}{2}$, 24 cents. **20.** $a+y+12$, 30 cents. **21.** $4y+c+5$, \$49. **22.** $10b+cy$, \$1. **23.** $a+\frac{a}{2}+a+\frac{a}{2}$, \$24. **24.** $\frac{x+y}{2}$, 30 cents.

Equations.—Page 54.

$$
\begin{aligned}
\textbf{1.}\ 3x+\tfrac{1}{4}x&=45-6\\
\underline{36+3}&\underline{=39}\\
\tfrac{13}{4}x&=39\\
x&=12
\end{aligned}
$$

After finding the value of x, substitute, and prove the equation [1 − 6].

2. 27. **3.** 14. **4.** 18. **5.** 20. **6.** 8. **7.** \$3, hat; \$15, coat. **8.** \$2000, house; \$500, lot. **9.** 20 each. **10.** \$3, William; \$9, John. **11.** \$2.40. **12.** \$1, box raisins; \$.50, box figs. **13.** 10 cents, muslin; 40 cents, linen. **14.** \$15, third; \$10, second; \$20, first. **15.** 10 rd., width; 30 rd., length. **16.** 5 hr. **17.** 6 hr. **18.** 8 dimes; 24 nickels. **19.** 5 each. **20.** 18 rabbits, George; 6 rabbits, Frank; 12 rabbits, Thomas. **21.** \$12, Anna; \$24, Mary; \$14, Emma. **22.** \$75. **23.** \$20. **24.** \$44. **25.** \$10,000.

Subtraction.—Page 60.

1. 4. **2.** +1. **3.** −12 ft. **4.** +16. **5.** $+9a$. **6.** −11 cents. **7.** $+b$. **8.** $-9c$. **9.** $+4a$. **10.** $+13d$. **11.** $-13b$. **12.** $+13ac$. **13.** $-14a^2$. **14.** $+14x^2$. **15.** $+4y^2$. **16.** $+13ab^2$. **17.** $-16a^2x$. **18.** $+12a^2c^2$. **19.** $+5ax$. **20.** $-2a^2b$. **21.** $-4xy^2$. **22.** $-4b^2x$. **23.** $-5a^2y^2$. **24.** $+6b^2c^3$. **25.** $4a$. **26.** $-9b$. **27.** 0. **28.** $-2c-5b$. **29.** $2xy$. **30.** $-6c^3$. **31.** $-b^2$. **32.** $4a$. **33.** $-\frac{3}{4}x^2$. **34.** $6a$. **35.** $-13b$. **36.** $10c$. **37.** $-5x$. **38.** $-16y^2$. **39.** $2a^2b$. **40.** $-b^2x^2-2a^2b^2$. **41.** $5a^3$. **42.** $-2a^2xz^3$. **43.** $2x$. **44.** $y+9$. **45.** $2+15xy$. **46.** $-3ax-8bc$. **47.** $8a+b$. **48.** $-3ax+30$. **49.** $5a^2x-3y^3$. **50.** $2\frac{1}{2}ab-1\frac{1}{4}c^3$. **51.** $2b$. **52.** $-7b+3c$. **53.** $10ax-2y$.

54. $-11x^2-3ay-4y^3$. **55.** $6a^2x+15$. **56.** $11b^2c-6a^3$. **57.** $-2x+2y-3z$. **58.** $17a^2x-11a^2y-a$. **59.** $-3ax^2-9ax+4a$. **60.** $-4a^2bc+5ay-16$. **61.** $8de-4f+6$. **62.** $12a^2xy+4a-9$. **63.** $2ab^2-5x^2y-5a^2b$. **64.** $7x+11ay-6$. **65.** $-14ay^2-13x^2y^3-3x^3$. **66.** $a^2x+8c-1$. **67.** $-11bxy+3c$. **68.** $-10x^3+14x^2y+3y^2$. **69.** $-7cy+3a^2x-6cd^2$. **70.** $5+17x^2+2a$. **71.** $2h-7c^2d+14a^2$. **72.** $-2\frac{1}{2}b^2c^3-\frac{1}{2}a^2b$. **73.** $-12a-2\frac{1}{2}x^2y^3-7$. **74.** $-3y^3+\frac{1}{4}a^2x-\frac{1}{2}$. **75.** $(a-b)c$. **76.** $(b-1)x$. **77.** $(-2+a)b$, or $(a-2)b$. **78.** $(a+4)c$. **79.** $(1+c)d$. **80.** $(-a+b)y$. **81.** $3y$. **82.** $(-3c-b)d$. **83.** $3bc$. **84.** $-3a^3$. **85.** $(3-a)(x+1)$. **86.** $(-1-a)(a+x)$. **87.** $7b+3c$; $-6a+11b+5c$. **88.** $-a-11y+3x$; $-5a-14y+10x$. **89.** $-12d-2b+c$; $10b-5c+20d$. **90.** $-2cd-3bc+ab$; $-7cd+bc-4ab$. **91.** ax^2-by; $-5a^2b+3by-9ax^2$. **92.** bc^2-7cy^2+4bx; $-bc^2-6cy^2-6bx$. **93.** $-8c^2d+9xy^2+4a^2x$; $-8c^2d+4xy^2+7a^2x$. **94.** $3\frac{1}{4}dx+2\frac{1}{2}cy^2-\frac{1}{4}xy^2$; $1\frac{1}{4}dx+3cy^2-\frac{1}{2}cy^2$. **95.** $2\frac{1}{3}a^2b+5\frac{2}{3}c^2x^2+\frac{1}{4}bc^2$; $2a^2b+5\frac{2}{3}c^2x^2+\frac{3}{4}bc^2$. **96.** $\frac{1}{4}b+2\frac{1}{3}c+\frac{1}{2}a$; $4b+\frac{3}{4}a+\frac{1}{3}c$; $3\frac{3}{4}b+\frac{1}{4}a-2c$. **97.** a^2b+2a^3-4bc; $a^2b+\frac{1}{2}bc+5a^3$; $-3a^3-4\frac{1}{2}bc$.

Changing Signs of Each Subtrahend (90–97).

90. $3cd-7bc+6ab$. **91.** $-5a^2b+by-7ax^2$. **92.** bc^2-7cy^2+4bx; $3bc^2-8cy^2+14bx$. **93.** $-8c^2d+9xy^2+4a^2x$; $-8c^2d+14xy^2+a^2x$. **94.** $3\frac{1}{4}dx+2\frac{1}{2}cy^2-\frac{1}{4}xy^2$; $5\frac{1}{4}dx+2cy^2$. **95.** $2\frac{1}{3}a^2b+5\frac{2}{3}c^2x^2+\frac{1}{4}bc^2$; $2\frac{2}{3}a^2b+5\frac{2}{3}c^2x^2-\frac{1}{4}bc^2$. **96.** $4b+\frac{3}{4}a+\frac{1}{3}c$; $4\frac{1}{4}b+1\frac{1}{4}a+2\frac{2}{3}c$. **97.** a^2b+2a^3-4bc; $2a^2b+7a^3-3\frac{1}{2}bc$.

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98. $a+b$. **99.** $x-y$. **100.** $+10$ dollars.

Algebraic Expression.—Page 63.

$a=5,\ b=3,\ c=2,\ x=50,\ y=10.$

1. $a-b$, 2. **2.** $3b-\frac{1}{2}c$, 8. **3.** a^2-y, 15. **4.** $2x-\frac{1}{4}a^2$, $93\frac{3}{4}$. **5.** $ac-\frac{2}{5}y$, 6; $3b-\frac{3}{4}c$, $7\frac{1}{2}$. **6.** $\frac{3}{4}by-c^3$, $14\frac{1}{2}$. **7.** $3b+a-\frac{4c}{2}$, 10. **8.** $\frac{a+b+y}{c}$, 9. **9.** $4b^2-\frac{c^3}{4}$, 34. **10.** $6ac-2b^2$, 42. **11.** $x-y$, 40 marbles. **12.** $4a-y=10$, 10.

13. $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

$\frac{1}{6}$ my money $=a$

My money $=6a$

$5\times 6=\$30$

14. $y+4+c$, 16 yr. **15.** $2y+2y-2b$, 34 in. **16.** $a-1$, 4; $a-3$, 2. **17.** $x-y$, 40 cents; $x+x-y$, 90 cents. **18.** $ac-b$, 7 cents. **19.** $x-\frac{1}{5}x$, \$40. **20.** $5y-\frac{1}{4}y$, \$47.50. **21.** $abc-y$, 20 cents. **22.** $x-4a-y$, \$20. **23.** $x-y-4b$, 28 miles. **24.** $a+b$, 8 miles. **25.** $ab-bc$, 9 cents.

Equations.—Page 65.

$$1.\quad x-\frac{x}{3}=25+5$$

$$\frac{45-15=30}{}$$

$$\tfrac{2}{3}x=30$$

$$x=45$$

After finding the value of x, substitute, and prove the equation.

2. 16. **3.** 12. **4.** 14. **5.** 10. **6.** 14. **7.** 22. **8.** 6 cents. **9.** 6 miles. **10.** \$240, horse; \$160, carriage. **11.** \$1200. **12.** \$12. **13.** 16 pigeons. **14.** 12 oranges. **15.** 9 bills of each, \$63. **16.** \$72. **17.** 18 in. long; 12 in. wide. **18.** \$5000. **19.** \$9. **20.** 40 papers, Monday; 30, Tuesday. **21.** 60 oranges, Frank; 40, Ned. **22.** 30 cents, Mary; 20 cents, Anna. **23.** 8 marbles, Harry; 24, William. **24.** \$125, horse; \$75, carriage; \$25, harness. **25.** \$15, man; \$10, boy.

Parentheses.—Page 68.

1. 9. **2.** 7. **3.** 8. **4.** 2. **5.** 5. **6.** 45. **7.** 12. **8.** 3. **9.** 7. **10.** 12. **11.** 3. **12.** 7. **13.** 16. **14.** $40-6+2=32$. **15.** 3. **16.** 18. **17.** 24. **18.** $18-6=12$. **19.** 30. **20.** 30. **21.** $1+2=3$. **22.** 12. **23.** 2. **24.** 6.

$a=8,\ b=4,\ c=3,\ d=2.$

25. $4+3=7$. **26.** $8-7=1$. **27.** $12-1=11$. **28.** $9-2=7$. **29.** $4+4=8$. **30.** $16-2=14$. **31.** $9+2=11$. **32.** $16\div 2=8$. **33.** $61\div 2=30\frac{1}{2}$. **34.** $16\times 1=16$. **35.** $2\times 6=12$. **36.** $2+2=4$. **37.** $10\div 4=2\frac{1}{2}$.

$a=8,\ b=6,\ c=4,\ d=2.$

38. $a-(b+d)$, 0. **39.** $(a-b)+c$, 6. **40.** $(a+b)-cd$, 6. **41.** $(b-c)+bc$, 26. **42.** $(a-b)\times(b+c)$, 20. **43.** $20-2(c+d)$, 8. **44.** $[(a+2b)-c]\times 4$, 64. **45.** $\frac{a}{c}+abc$, 194. **46.** $4bc+\frac{c}{d}$, 98. **47.** $\frac{6a}{d}-(b+$

$c)$, 14. **48.** $2bc-\frac{a}{c}$, 46. **49.** $\frac{6c}{c+d}+bc$, 28. **50.** $2a-b+\frac{a+c}{b}$, 18.
51. $[4ab+(a-c)]-2a$, 180. **52.** $4(a+10)$, 72 marbles.

Page 72.

1. 6. **2.** 26. **3.** 4. **4.** 5. **5.** 8. **6.** 1. **7.** 3½. **8.** 12.
9. 0. **10.** −12. **11.** −8. **12.** 6. **13.** −18. **14.** −14.
15. −10. **16.** −6. **17.** −13. **18.** −23. **19.** −17. **20.** 7.
21. −13. **22.** 0. **23.** 4. **24.** −6. **25.** −6. **26.** 10. **27.** 6.
28. 12. **29.** 12. **30.** 8. **31.** 22.

$a=4,\ b=3,\ c=2,\ d=1,\ x=15,\ y=10,\ z=5.$

32. $a+b-c-d$, 4. **33.** $a-b-c+d$, 0. **34.** $a-b+c-d$, 2. **35.** $a+b+c-d$, 8. **36.** $11a-3a$, 32. **37.** $8a-6a$, 8. **38.** $7x-7x$, 0.
39. $8x-6x$, 30. **40.** $8x-6y$, 60. **41.** $8x-6y$, 60. **42.** $2y-2z$, 10.
43. $10y-2b$, 94.

$a=8,\ b=6,\ c=5,\ x=4,\ y=3,\ z=2.$

44. $(a+b+c)x$, 76. **45.** $(a+c)y+(a-4)z$, 47. **46.** $(10-a)x+(b-c)y$, 11. **47.** $(a+b)x\times 4$, 224. **48.** $(a-c)y\div 3$, 3. **49.** $6\times(b-c)x$, 24.

Expressing Terms as Binomials.

$a=4,\ b=3,\ c=2,\ x=10,\ y=5.$

$$\begin{aligned}&\textbf{50.}\ (2x-3a)+(4b-5c)\\&(20-12)+(12-10)\\&8+2=10\end{aligned}$$

51. $15-2=13$. **52.** $25-20=5$. **53.** $5-1=4$. **54.** $50-13=37$.
55. $5-6=-1$.

Expressing First Terms as Trinomials.

$$\begin{aligned}&\textbf{50.}\ (2x-3a+4b)-5c\\&(20-12+12)-10\\&20-10=10\end{aligned}$$

51. $7+6=13$. **52.** $13-8=5$. **53.** $13-9=4$. **54.** $41-4=37$.
55. $-3+2=-1$.

Expressing Last Terms as Trinomials.

50. $$\left.\begin{array}{l}2x-(3a-4b+5c)\\20-(12-12+10)\\20-10=10\end{array}\right\}\text{ Or, }\begin{array}{l}2x+(-3a+4b-5c)\\20+(-12+12-10)\\20+(-10)=10\end{array}$$

51. $10+3=13$. **52.** $30+(-25)=5$. **53.** $10+(-6)=4$. **54.** $40+(-3)=37$. **55.** $20+(-21)=-1$.

56. (May be arranged in an indefinite number of ways, with a corresponding number of values, the final results being identical.)

Transposition in Equations.—Page 77.

1. $$\begin{array}{c}x+5=11\\ \underline{6+5=11}\\ x=6\end{array}$$

Find the value of x, substitute, and prove the equation.

2. 9. **3.** 5. **4.** 6. **5.** 6. **6.** 7. **7.** 3. **8.** 5. **9.** 2.

10. $$\begin{array}{c}4x=3x+6+2\\32=24+6+2\\x=8\end{array}$$

11. 1. **12.** 6. **13.** 4. **14.** 6. **15.** 10. **16.** 12. **17.** 3.
18. 6. **19.** 12. **20.** 8.

21. 1. **22.** 3. **23.** 6. **24.** 19. **25.** 43. **26.** 12. **27.** 5.
28. 5. **29.** 9. **30.** 12.

In problems 21–30, y, when substituted for x, will have the same value as x.

31. 69. **32.** 5. **33.** 21. **34.** 8. **35.** 7. **36.** 12. **37.** 8.
38. 2. **39.** 8. **40.** 12.

In problems 31–40, z, when substituted for x, will have the same value as x.

41. 5. **42.** 25. **43.** 9. **44.** 9. **45.** 6. **46.** 9. **47.** \$18, A's; \$22, B's. **48.** \$12.50, chain; \$37.50, watch. **49.** 8 yr. **50.** 10 marbles. **51.** 12. **52.** \$5, William; \$15, Thomas. **53.** 12 ft. **54.** 3yr., John; 15 yr., George. **55.** 6, first; 24, second; 60, third. **56.** \$14, George; \$15, Charles; \$18, John. **57.** 30 pigeons, Fred; 20, Harry; 5, Robert.

58. 30, greater; 13, less. **59.** 20 oranges, Mary; 5, Mabel. **60.** 3 ft. in water; 18, in air. **61.** 24 marbles, Fred; 9, William; 75, Harry. **62.** \$89. **63.** \$1000. **64.** \$51.

65. Let $x =$ number acres,

Then $\frac{3x}{8}+10 =$ number acres sold,

And $\frac{5x}{8}-10=\frac{x}{2}-4$

$$\begin{array}{c}\hline \frac{x}{8}=6 \\ x=48 \text{ acres.}\end{array}$$

66. 1560 persons for successful candidate; 1190, for unsuccessful candidate. **67.** \$2 daily, father; \$1.50, first son; \$1, second son. **68.** \$3, first day; \$4, second; \$7, third. **69.** \$100. **70.** 50 cents, first; 75 cents, second.

Multiplication.—Page 84.—Monomials.

$a=1,\ b=2,\ c=3,\ d=4,\ x=5,\ y=6.$

1. 42. **2.** −42. **3.** −42. **4.** 42. **5.** $6x$, 30. **6.** $-6x$, −30. **7.** $-6x$, −30. **8.** $6x$, 30. **9.** $24abc$, 144. **10.** $-12a^2y$, −72. **11.** $-5ax$, −25. **12.** a^2b^2, 4. **13.** $7a^3bc$, 42. **14.** $-8b^2x^2$, −800. **15.** $-5a^2b$, −10. **16.** $6a^4$, 6. **17.** $-6x^2y^2$, −5400. **18.** $-6abc^2d$, −432. **19.** $12a^2bcd$, 288. **20.** $20ab^3cd^2$, 7680.

21. $-21b^2c^3dx^2$. **22.** $-36a^3bc^2d^2$. **23.** $33b^3c^2dx^2y$. **24.** $32a^2b^2cx^2$. **25.** $-24a^2bx^3y^2z$. **26.** $-40b^2c^3u^2v^3$. **27.** $60c^2d^3y^3z^2$. **28.** $15a^2bv^2wx^3$. **29.** $-35b^2c^3w^3x^3y$. **30.** $-63ac^2xy^2z^3$. **31.** $18a^3b^2y^3z^2$. **32.** $24a^5b^2$. **33.** $-30b^3c^5$. **34.** $-28c^5d^4$. **35.** $18d^5c^2e^4$. **36.** $-50x^4y^3$. **37.** $-32a^3b^2x^5$. **38.** $36b^4x^5y$. **39.** $12a^5xy^3$. **40.** $-24b^3x^4y^2$. **41.** $-32c^3x^3y^5$. **42.** $20ax^3y^2z^3$. **43.** $-12x^2y^2z^5$. **44.** $-36a^3b^3z^4$. **45.** $50c^7d^2$. **46.** $-120a^5bc$. **47.** $-16a^2b^4c^3$. **48.** $-24a^5x^4y$. **49.** $-8a^4b^5c^4$. **50.** $50a^5x^5y$. **51.** $24a^7xy^2$. **52.** $-16a^2b^6x^3$. **53.** $-108a^5b^4y$. **54.** $32b^3x^3y^2$. **55.** $-6b^3c^5x^3$. **56.** m^4. **57.** a^4. **58.** a^4x^4. **59.** $36a^2x^2$. **60.** $-4a^4x^3$.

A Polynomial by a Monomial.—Page 86.

61. $20a+8b$. **62.** $-6a^3x+2a^2y^2$. **63.** $8a^4b^3-10a^2b^3c^2-2a^2b^2y$. **64.** $12a^3bc-6ac^3$. **65.** $-10ax^2y+6x^2y^3$. **66.** $9b^3c^3+3a^2b^2c^2--6b^2c^5$.

67. $-20a-8x+12y$. **68.** $8ax^3y-3a^2x^2y+x^4y$. **69.** $20a^5b^2-25a^4b^3-35a^3b^5$. **70.** $35a^2b^3c-49a^4b^2c-84abc$; $-15a^3b^3+21a^5b^2+36a^2b$. **71.** $-49a^3c^3+56a^4c^2-63a^5c^5$; $42ac^3d-48a^2c^2d+54a^3c^5d$. **72.** $-45a^3b^4c+40a^3b^4+35a^2b^3$; $90ab^3c^4-80ab^3c^3-70b^2c^3$. **73.** $-14a^4y^2+10a^3y^3-6a^2y^4$; $21a^5y^2-15a^4y^3+9a^3y^4$. **74.** $-20a^2x^3y^2+32a^3x^2y^2+8a^4y^3$; $10a^3x^3y-16a^4x^2y-4a^5y^2$. **75.** $18x^6y^2+12x^4y^4-15x^2y^6$; $-24x^5y^3-16x^3y^5+20xy^7$. **76.** $-30a^2b^3x+42a^2b^2x^2-54a^4b^2$; $20ab^4x-28ab^3x^2+36a^3b^3$. **77.** $7a^5b^2x^2+5a^4bx^3-3a^3x^4$; $-14a^4b^2x^3-10a^3bx^4+6a^2x^5$. **78.** $24a^3x^5y^3+15a^4x^3y^3+3ax^3y^4$; $-32a^3x^5-20a^5x^3-4a^2x^3y$. **79.** $-27a^6b^3c^5+45a^4b^4c^5+63a^2b^3c^5$; $18a^5b^2y^3-30a^3b^3y^3-42ab^2y^3$. **80.** $27c^5dx^2-21c^3x^3+3c^2x^5$; $-36c^4dx^3+28c^2x^4-4cx^6$. **81.** $-6x^4y^3z^2+12x^3y^2z^3+18x^2yz^4$; $10x^5y^5z-20x^4y^4z^2-30x^3y^3z^3$. **82.** $24a^6x^4y^3+12a^4x^2y^4-6a^2xy^5$; $-28a^7x^5y-14a^5x^3y^2+7a^3x^2y^3$. **83.** $-42a^6bm^2-84a^3bm^4+28a^2bm^6$; $30a^6m^2+60a^4m^4-20a^3m^6$. **84.** $36a^3c^2-16a^2c^3+42ac^4$; $-18a^5c+8a^4c^2-21a^3c^3$. **85.** $16a^2(a-b)+20a^2(a+b)-2a^2c$; $-32b(a-b)-40b(a+b)+4bc$. **86.** $-6a^5y^3+10a^4x^3-6a^3y^2$; $12a^2x^2y^6-20ax^5y^3+12x^2y^5$. **87.** $12a^2d+a^2bc^2d^3-3a^3c^3d^6$; $-48a^3c-4a^3bc^3d^2+12a^4c^4d^6$. **88.** $-20ax^2y^3+8ax^4y^2-20a^2xy^4$; $5a^3x^3y^2-2a^3x^5y+5a^4x^2y^3$. **89.** $35a^5m^3-25a^4m^4+15a^3m^5$; $-28a^7m+20a^6m^2-12a^5m^3$. **90.** $-10x^5y^3+3x^2y^5z-12x^4y^2z^2$; $20x^4y^3-6xy^5z+24x^3y^2z^2$.

A Polynomial by a Polynomial.—Page 88.

91. $2a^2+5ab+3b^2$.
92. $3a^2-5ab+2b^2$.
93. $6x^2+xy-12y^2$.
94. $8x^2-32y^2$.
95. $8b^2-18c^2$.
96. $3x^2+xy-4y^2$.
97. $6x^4+13x^2y^2+6y^4$.
98. $12a^2+25ax+12x^2$.
99. $ac+bc-ad-bd$.
100. $8a^2+10ab+3b^2$.
101. $8x^2+10xy+3y^2$.
102. $3a^2+4ab-4b^2$.
103. $a^3+ab^2+a^2b+b^3$.
104. $-x^2+x+20$.
105. $-5-7a+6a^2$.
106. $2m^2-mn-3n^2$.
107. $8y^2-14yz+6z^2$.
108. $6a^2+3a-30$.

109. $x^3+2xy+3x^2y^2+6y^3$. **110.** $6a^2b^2-11abc+4c^2$. **111.** $-4c^2d^2-10cd^2+6d^2$. **112.** $m^2+4m-60$. **113.** $1+c-a^2-ac$. **114.** $x^2-xy-y-1$. **115.** $a^2+ab-bc-c^2$. **116.** $xy-y^2-xz+z^2$. **117.** $4a^2+2ab+bc-c^2$. **118.** $8a^2y^2-72y^2$. **119.** $a^2+ab+bc-c^2$. **120.** $a^3+a^2b-ab^2-b^3$. **121.** $x^3-x^2y-xy^2+y^3$. **122.** a^6-a^2. **123.** a^3+x^3. **124.** x^3-y^3.

125. x^2-y^2+2y-1. **126.** $x^2-6x-40$; x^2+6x. **127.** $4a^2-25$; $10a-25$. **128.** $36-42a+10a^2$; 4. **129.** $x^2-2x-15$; $15+2x-x^2$. **130.** a^2-b^2; a^4-b^4. **131.** x^3+y^3; $2xy$. **132.** $4x^2-10x-50$; $8x^2-100$; $-100-5y^2$. **133.** a^2-9x^2; $2a^2-6ax$; $2a^2-2x^2$.

134. $4a^2$; $4a^4-8a^3b+4a^2b^2$; $4a^4-4a^2b^2$. **135.** $4c^2-2$; $16c^4-4$; $32c^4-8c^2$. **136.** a^3+b^3; $-2a^2b^2$; $-2b^3+2a^2b^2$. **137.** $4a^2-4b^2$; $-8ab-8b^2$; $12a^2-12b^2$. **138.** b^3-c^3; $2b^3-2b^2c^2$; $-2b^3+2b^2c^2$. **139.** x^4-y^4; $2x^4+2x^2y^2-2y^4$; $2x^2y^2+2y^4$. **140.** a^2-4a+4; a^2+4a+4; $-8a$; $8a^2-24a+8$.

Expanding Algebraic Expressions.—Page 92.

1. $b^2+2bc+c^2$. **2.** $c^2+2cd+d^2$. **3.** $a^2+2ax+x^2$. **4.** $a^2+2ay+y^2$. **5.** $x^2+2xy+y^2$. **6.** $a^2+2az+z^2$. **7.** $9+6b+b^2$. **8.** x^2+4x+4. **9.** $9+6y+y^2$. **10.** $4a^2+4ab+b^2$. **11.** $b^2+6bc+9c^2$. **12.** $c^2-2cd+d^2$. **13.** $m^2-2mn+n^2$. **14.** $a^2-2am+m^2$. **15.** $b^2-2bn+n^2$. **16.** $c^2-2cx+x^2$. **17.** $x^2-2xy+y^2$. **18.** $4+4a+a^2$. **19.** $9a^2-6ax+x^2$. **20.** $4a^2+12ac+9c^2$. **21.** b^2-c^2. **22.** c^2-d^2. **23.** a^2-x^2. **24.** y^2-a^2. **25.** $x^4+2x^2y+y^2$. **26.** $y^2-2yz+z^2$. **27.** a^2-4. **28.** $9-a^2$. **29.** x^2-1. **30.** $4a^2-x^2$. **31.** b^2-9c^2. **32.** $4a^2b^2-c^2$. **33.** $4-b^2c^4$. **34.** a^4-x^4. **35.** $9-y^4$. **36.** $4b^2c^2-16$. **37.** $4-9x^4$. **38.** a^6-b^6. **39.** $16a^2-36$. **40.** $4x^4-25$. **41.** $a^2+2ab+b^2$. **42.** $a^2-2ac+c^2$. **43.** $4+4b+b^2$. **44.** b^2-6b+9. **45.** $a^2+4ax+4x^2$. **46.** $4a^2-4ax+x^2$. **47.** $y^2+4cy+4c^2$. **48.** $9-6z+z^2$. **49.** z^2+6z+9. **50.** $4a^2-4ac+c^2$. **51.** $a^2-4ac^2+4c^4$. **52.** $4a^2-12ac+9c^2$. **53.** $9c^2-12ac+4a^2$. **54.** $a^4+2a^2x^2+x^4$. **55.** $x^4-2x^2y^2+y^4$. **56.** $25+10ax+a^2x^2$. **57.** $a^2y^2-12ay+36$. **58.** $4a^4+4a^2+1$.

Page 93.

61. $a^2+7a+12$. **62.** $b^2+9b+20$. **63.** c^2+5c+6. **64.** $d^2+8d+15$. **65.** $x^2+8x+12$. **66.** $x^2+10x+21$. **67.** $4+7(2)+12$. **68.** a^4+10a^2+24. **69.** $a^2x^2+8ax+15$. **70.** $4x^4+8x^2+3$. **71.** a^2-5a+6. **72.** $b^2-9b+20$. **73.** $c^2-9c+18$. **74.** $d^2-11d+28$. **75.** $x^2-7x+10$. **76.** $y^2-9y+20$. **77.** $16-5(4)+6$. **78.** x^4-9x^2+20. **79.** $a^2x^2-7ax+10$. **80.** $4a^4-8a^2+3$. **81.** b^2-b-12. **82.** $c^2-4c-12$. **83.** $a^2-4a-21$. **84.** $x^2-2x-24$. **85.** $y^2-2y-35$. **86.** $z^2+2z-15$. **87.** $a^2x^2+3ax-40$. **88.** $25+2(5)-8$. **89.** a^4+2a^2-24. **90.** $9x^4-3x^2-20$. **91.** $a^4+5a^2b+6b^2$. **92.** $9+7(3)+10$. **93.** $x^2-6ax+8a^2$. **94.** $25-5(5)+6$. **95.** $9x^2-9x-4$. **96.** $4a^2x^4-2ax^2-20$. **97.** $49+7a^2b-6a^4b^2$. **98.** $4a^2b^2+24ab+35$. **99.** $25-8ax-5$. **100.** $36x^4y^4-12x^2y^2-35$.

Exercises in Algebraic Expression.—Page 94.

$a=8,\ b=6,\ c=4,\ x=50,\ y=10.$

1. a^2, 64. **2.** a^2b^3, 13,824. **3.** $5c^3$, 320. **4.** $6ay$, 480. **5.** $10b^2c^3$, 23,040. **6.** $5c-\frac{x}{10}$, 15. **7.** $2by$, \$1.20. **8.** $bc+y$, 34 miles. **9.** $6b$, 36 days. **10.** $(32b+c)$ qt., 196 qt.

$$\begin{aligned} \textbf{11.}\ &(a+3)(a-2) \\ &a^2+a-6 \\ &64+8-6=66. \end{aligned}$$

12. $12y-2x$, 20 cents. **13.** $c(b+1)-bc$, 4 mi. **14.** $a+3a-8$, 24 cows and sheep, 16 sheep. **15.** $10a+b$, 86. **16.** $(ab)-b^2$, 12 sq. in. **17.** $2x-(8b+8c)$, \$20. **18.** $\frac{bcx}{100}$, \$12. **19.** $b+\frac{1}{20}b$, \$6.30. **20.** $x-\frac{1}{50}x$, \$49.

Equations.—Page 95.

$$\begin{aligned} \textbf{1.}\ 3(x+2)&=7(x-2) \\ 3x+6&=7x-14 \\ \underline{15+6}&\underline{=35-14} \\ -4x&=-20 \\ x&=5 \end{aligned}$$

In problems 2–12 find the value of x, substitute, and verify each equation.

2. 4. **3.** 3. **4.** 5. **5.** 5. **6.** 6. **7.** 3. **8.** 3. **9.** 10. **10.** $30-11x+x^2=56-15x+x^2-10$, 4. **11.** $3x-12+12x=11x+16$, 7. **12.** $x^2-23x+60=x^2-5x-84$, 8.

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13. 45, 15. **14.** 6, less; 30, greater. **15.** 3. **16.** 8. **17.** 25 cents. **18.** 48 yd.

19. Let $x=$ number of years,

$$\begin{aligned} \text{Then}\ \underline{28+x}&\underline{=3(4+x)} \\ 28+x&=12+3x \\ -2x&=-16 \\ x&=8\ \text{years.} \end{aligned}$$

20. 25 yr. **21.** \$12. **22.** \$1.00, son; \$2.25, father. **23.** \$12, A; \$28, B. **24.** 2 cents, pear; 3 cents, lemon; 5 cents, orange.

25. Let $x = \text{side}$,

Then $x + 4 = \text{side increased}$,

And $\underline{(x+4)^2 = x^2 + 80}$ sq. in.

$$x^2 + 8x + 16 = x^2 + 80$$
$$8x = 64$$
$$x = 8$$

26. 20 in., length; 5 in., width. **27.** 192. **28.** 5 pigeons, George; 25, Frank; 15, William. **29.** \$800 @ 6%; \$1600 @ 5%. **30.** 4 hours. **31.** 12 in., length; 6 in., width. **32.** 25 papers. **33.** 30 cents, James; 20 cents, John; 100 cents, Robert. **34.** 40 marbles, Thomas; 30, Frank; 60, George.

35. Let $x = \text{distance}$,

Then $\frac{x}{9} = \text{number hours going}$,

And $\frac{x}{6} = \text{number hours returning}$,

Hence, $\underline{\frac{x}{9} + \frac{x}{6}} = 5$ hours.

$$\frac{5x}{18} = 5$$
$$x = 18 \text{ miles.}$$

Division.—Page 100.—A Monomial by a Monomial.

1. 3. **2.** -3. **3.** $-3a$. **4.** $3x$. **5.** $-3y$. **6.** 7. **7.** 3. **8.** 2. **9.** $-2c$. **10.** 2. **11.** a. **12.** c. **13.** $-3x^4$. **14.** -6. **15.** $2a$. **16.** a^3x^3. **17.** $-11xy^2$. **18.** $3yz$. **19.** $3a^2c^2$. **20.** $5ay$. **21.** $4x^4y^4$. **22.** $-3c^2$. **23.** $-bc$. **24.** $-a^3y^2$. **25.** $4xyz^4$. **26.** $-7ac$. **27.** $3b^3d^2x^5$. **28.** $4cf^3$. **29.** $2am$. **30.** $7xy$. **31.** $-2m^4x^7$. **32.** $-2a^6z$. **33.** $9x^6y^4$. **34.** $-4bx^5$. **35.** -1. **36.** $-8m^2n^3$. **37.** $-4x^4$. **38.** $-9wz$. **39.** $-5y^7z^3$. **40.** $3a^2c$. **41.** $-4n^3$. **42.** $-3n^4x$. **43.** $2b^4c$. **44.** $-3b^2cd^4$. **45.** $-4x^4y$. **46.** ad^5. **47.** $-d^7x^4y$. **48.** $-7c^8d$. **49.** $-5x^4z$. **50.** $-4bd$.

A Polynomial by a Monomial.—Page 102.

51. $a+2b$. **52.** $2x-3$. **53.** $-2x+3$. **54.** $-a+3x^2$. **55.** $a-a^2$. **56.** $-bc+b^2x$. **57.** $-c^2-d^3$. **58.** $x-2x^2y$. **59.** $7b-c$. **60.** $2c-4b$. **61.** $-2+xz$. **62.** $y-2z$. **63.** $-b^2x^2+y^2$. **64.** $1+5yz^2$.

65. $-a^3c+2a^2$. **66.** $-3a-4b^2$. **67.** $-3c+9b^2$. **68.** $-2xy+3$. **69.** $-2x^3+3abx^2$. **70.** a^2+2a+1. **71.** $-x^2+2xy+y^2$. **72.** $y+3x-xy^3$. **73.** $-a+b-c$. **74.** $-x-1+xy$. **75.** $-(a+x)+2(a+x)$. **76.** $-\overline{a-x}+2\overline{a-x}$. **77.** $-x^2+2xy-y^2$. **78.** $3b^2-2bc+c^2$. **79.** $-2x^2+3ax-4a^2$. **80.** $5y^2+4xy-3x^2$. **81.** $3x^4-2x^2y-y^2$. **82.** $5a^2-3ac+2c^2$. **83.** $-3x^3+2a^2x^2-4a^4x$. **84.** $4c+3ab-2b^2c^2y$. **85.** $-5c+3ady-4ey^3$. **86.** $-x-2y-z$. **87.** $-2+3am-4a^2m^2$. **88.** $-a^2+2a+3$. **89.** $-2(a+x)+3(a+x)-(a+x)$. **90.** $+a(a-c)-2b(b+c)$. **91.** $-1+2a+3b$. **92.** $-2x+3+1$. **93.** $-1-2cx+3c^2x^2$. **94.** $-2x^2-3px-4p^2$. **95.** $3r^2-2rst+s^2t^2u$. **96.** $-3b^2z+2abyz-a^2y^2z^2$. **97.** $-m^2ox^3+2amo^2x^2-3a^2o^3x$. **98.** $-a+2b-3$. **99.** $\frac{1}{4}b^2+\frac{1}{3}abd-\frac{1}{6}a^2$. **100.** $-a+2a^3b-3ab^2$.

A Polynomial by a Polynomial.—Page 105.

101. c. **102.** z. **103.** $a+b$. **104.** $b-c$. **105.** $a-4$. **106.** $x+2$. **107.** $a+2c$. **108.** $a-3$. **109.** $a+4$. **110.** $a+3$. **111.** $x+2$. **112.** $7+x$. **113.** $x-10$. **114.** a^2+ab+b^2. **115.** x^2-x+1. **116.** $a+x$. **117.** $b-c$. **118.** x^2+3. **119.** y^3-4. **120.** x^2-y^2. **121.** $m+n$. **122.** $r+s$. **123.** $2a^2x^2+1$. **124.** $m-n$. **125.** $4a^2-6ab+9b^2$. **126.** x^2-xy+y^2. **127.** a^2+ay+y^2. **128.** a^2-a+1. **129.** $x-3$. **130.** $x+1$. **131.** $3x-3y$. **132.** $3a+4$. **133.** $4a+2$. **134.** $2a^3-a^2b$. **135.** $2xy+2z$. **136.** $3a^2b-5c$. **137.** $-2ax+5y$. **138.** $5r^2s+6s$. **139.** x^3+y^3; $x+y$. **140.** $c-d$; $a-b$. **141.** $m-x-1$; $m+1$. **142.** b^4+b^2+1. **143.** $-1+x^2$. **144.** $x^6+4x^4+16x^2+64$. **145.** $2a+4$.

Inasmuch as the dividends in problems 101–145 are the products of the several divisors and quotients, it is not deemed necessary to insert them here.

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146. $2a^2+2b^2$; $2a^2-2b^2$. **147.** a^2-ax+x^2; $2ax$; $4a^3x-4ax^3$. **148.** $2c+1$; $8c^3+4c^2+2c+1$; $4c^2+1$. **149.** a^2+2a+1; $a+1$; $8a^4+8a^3-a-1$. **150.** x^2+xy+y^2; $+3xy$; $9x^3y^2-6x^2y+3xy$. **151.** x^3-y^3; $-2y^3$; $4y^4-y^2$. **152.** $1-2x^2-x^3$; $1-x-x^2$; $2-4x^2-2x^3$. **153.** a^4+a^2+1; $2a^4$; $+2a^2-1$.

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154. $b+c$. **155.** $x-y$. **156.** $a-1$. **157.** $2a+2c$. **158.** $3x-3b$. **159.** $4ac+2y$. **160.** $b+c$. **161.** $a-y$. **162.** $2a+1$. **163.** $3x^2-2y$. **164.** $2a^2b+2$. **165.** $3x^2y-2z$. **166.** b^2-bc+c^2.

167. a^2+ax+x^2. **168.** $4c^2-2c+1$. **169.** $9x^2+6x+4$. **170.** $4a^2x^2-2axy+y^2$. **171.** $16a^4+8a^2x^2+4x^4$.

The dividends in problems 154–171 are the several products required.

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172. $x-y+\frac{2y^2}{x+y}$. **173.** $a+x+\frac{2x^2}{a-x}$. **174.** $1-a-\frac{a^2}{1+a}$. **175.** $2c+8+\frac{39}{c-4}$; $2c^2+7$. **176.** $x+2+\frac{8}{x-2}$. **177.** $a-b+\frac{2b^2}{a+b}$. **178.** $x^2-xy+y^2-\frac{2y^3}{x+y}$. **179.** $c^2+cd+d^2+\frac{2d^3}{c-d}$. **180.** $b+3c+\frac{4c^2}{b-c}$. **181.** $x+2y+\frac{y^2}{x-y}$. **182.** $a-3b-\frac{2b^2}{a-b}$. **183.** $b-b^2+\frac{c}{ab^2}$.

Algebraic Expression.—Page 109.

$a=2$, $b=3$, $c=5$, $d=6$, $x=25$, $y=10$.

1. $\frac{a^2}{6}$, $\frac{2}{3}$. **2.** $\frac{b}{2}+c$, $6\frac{1}{2}$. **3.** $\frac{2y}{a}-b$, 7. **4.** $\frac{c^3}{a+b}$, 25. **5.** $\frac{2}{3}c+y$, $13\frac{1}{3}$. **6.** $\frac{6a^2}{d}$, 4. **7.** $\frac{bx}{c}$, 15 cents. **8.** $\frac{y-a}{4}$, 2. **9.** $\frac{ax+10}{a}$, \$30. **10.** $\frac{10b+a}{2}$, 16. **11.** $\frac{y}{b+2}$, 2. **12.** $\frac{c}{xy}$, 2 cents. **13.** $\frac{2x}{ac}$, 5 ft. **14.** $\frac{3x}{4c}$, $3\frac{3}{4}$ mi. **15.** $cd+b$, 33. **16.** $\frac{3x-b}{d}$, 12. **17.** $\frac{3x-b}{36}$, 2 yd. **18.** $\frac{1}{c}+\frac{1}{d}$, $\frac{11}{30}$. **19.** $\frac{y+a}{2}$, 6, larger; 4, smaller. **20.** $4\div\frac{2x}{25}$, 2 yr.

Examples and Problems.—Page 110.

1.
$$\begin{array}{c} \frac{1}{2}x+2=\frac{1}{4}x+4 \\ \underline{4+2=2+4} \\ \frac{1}{4}x=2 \\ x=8 \end{array}$$

In problems 1–30, the letters substituted for x will have the same value as x. Verify each equation.

2. 12. **3.** 18. **4.** 60. **5.** 20. **6.** 6. **7.** 12. **8.** 12. **9.** 12. **10.** \$50. **11.** 15 yr.

12. Let $x=$ miles per hour,

Then $\underline{6x=24+36}$ miles.

$6x=60$

$x=10$ miles per hour.

13. 6. **14.** 16, 8. **15.** 10 yr., Mary; 30 yr., mother. **16.** 40, 20. **17.** \$48, father; \$32, son. **18.** 60 cents, first; 40 cents, second; 30 cents, third. **19.** 12 yr. **20.** 8 5-dollar bills; 24 2-dollar bills.

21. Let $x = \text{distance}$,

Then $\frac{x}{3} + \frac{x}{6} = 6$ hours.

$$\frac{x}{2} = 6$$

$$x = 12$$

Therefore, he went 12 miles.

22. Let $x = \text{number of pigeons}$,

Then $\frac{x}{3} + 8 = \frac{2x}{3}$

$$\underline{8 + 8 = 16}$$

$$-\frac{x}{3} = -8$$

$$x = 24$$

Therefore, he had 24 pigeons, and they were worth \$6.

23. 7 kites @ 20 cents; 5 kites @ 12 cents. **24.** 20 marbles, James; 15, John. **25.** \$120. **26.** \$20, Emma paid; \$30, Ethel paid. **27.** \$500. **28.** 30 cents, first; 20 cents, second; 10 cents, third. **29.** \$4000 @ 5%; \$2000 @ 6%. **30.** \$400.

Factoring.—Page 114.

1. 2, 2, 2. **2.** 3, 3, 3. **3.** 5, 3, 3. **4.** a, b. **5.** b, b, x. **6.** 5, c, d, d. **7.** a, b, b, c. **8.** 5, 2, m, n, n, n. **9.** 7, 2, x, x, x, y, y. **10.** 2, 2, 3, y, z, z, z. **11.** -3, 3, a, a, x. **12.** -2, 7, a, z, z, z. **13.** -3, 3, 2, r, r, s, s, s. **14.** -2, 2, 5, y, y, y, z, z. **15.** -3, 3, 3, b, b, y, y, y.

In each instance the product of the factors is the original composite quantity.

In the following, *one* of the factors in each set meets the requirement of the problem:—

16. ab, ab. **17.** $2xy$, $2xy$. **18.** $3a^2b$, $3a^2b$. **19.** $4bc^2$, $4bc^2$. **20.** $5a^2b^3$, $5a^2b^3$. **21.** $\frac{1}{3}ax^2$, $\frac{1}{3}ax^2$. **22.** $\frac{1}{4}x^3z^4$, $\frac{1}{4}x^3z^4$. **23.** $6ax^3$, $6ax^3$. **24.** $3(a+b)$, $3(a+b)$. **25.** 2, 2, 2. **26.** 3, 3, 3. **27.** ab, ab, ab. **28.** $(x-y)$, $(x-y)$, $(x-y)$. **29.** $(a+x)$, $(a+x)$, $(a+x)$. **30.** $3ab^3$, $3ab^3$, $3ab^3$.

In each instance the product of the equal factors is the original square or cube.

31. $a, b+1$. **32.** $b, a+1$. **33.** $4, c+1$. **34.** $2x, x-2$. **35.** $3a, 1+x$. **36.** $4a, a^2-2$. **37.** $5a, b-5x$. **38.** $xy, 2x+y$. **39.** $3x, 2x-3a$. **40.** $4a^2, 2b-3c$. **41.** $7bc, c-2bd$. **42.** $5mn^2, m-3n$. **43.** $6x^2y^2, x-2y$. **44.** $18a^3, a^2x-1$. **45.** $(n+1), a-b$. **46.** $5, 2+3x-4x^2$. **47.** $3x^2, x^2-3x-2$. **48.** $7b^2, b+2b^2-3$. **49.** $6b, 1-2bc^2+3b^2c$. **50.** $4, x^2+2xy-3y^2$. **51.** $2ax^2, 3+4ax+8a^2x^2$. **52.** $y, 2x+3y+y^2$. **53.** $6a, a-5b+2d$. **54.** $bc, 4b+d-2e$. **55.** $7c, a^2-2ab+b^2$. **56.** $xy^2z^3, 1+xyz-x^2y^2z^2$. **57.** $a^2bc^2, a-b-c$. **58.** $xz, xyz^2+z-x^2y^2$. **59.** $\frac{1}{8}b, a-4c+2cd$. **60.** $\frac{1}{6}x^2y^3, 3y^2-2x^3yz+x^2z^2$.

In problems 31–60, the product of the factors is the composite quantity given.

To find Two Equal Binomial Factors of a Trinomial.—Page 116.

61. $a+b, a+b$. **62.** $b-c, b-c$. **63.** $x+1, x+1$. **64.** $1-x, 1-x$. **65.** $2c-3, 2c-3$. **66.** $2x+3y, 2x+3y$. **67.** $3n-2x, 3n-2x$. **68.** $a+2, a+2$. **69.** $3+c, 3+c$. **70.** $2y+2, 2y+2$. **71.** $3a-4, 3a-4$. **72.** $3c-3x, 3c-3x$. **73.** $2s^2+r, 2s^2+r$. **74.** $7-ab, 7-ab$. **75.** $3x^2-3, 3x^2-3$. **76.** $a-3, a-3$. **77.** $3a^2+b, 3a^2+b$. **78.** $2m^2+n, 2m^2+n$. **79.** $5x-1, 5x-1$. **80.** $1-5x, 1-5x$. **81.** $3bc-2, 3bc-2$. **82.** $r-s, r-s$. **83.** $3y-z, 3y-z$. **84.** $1+3x, 1+3x$. **85.** $2b-3, 2b-3$. **86.** $a^2b^2+c^2, a^2b^2+c^2$. **87.** $2-3a^2b, 2-3a^2b$. **88.** $1-2a^2b^3, 1-2a^2b^3$. **89.** $\frac{1}{2}a-b, \frac{1}{2}a-b$. **90.** $x+\frac{1}{3}, x+\frac{1}{3}$.

In problems 61–90, the products of the factors are the original composite quantities.

To find Two Binomial Factors of the Difference of Two Squares.—Page 117.

91. $a-x, a+x$. **92.** $b-c, b+c$. **93.** $m-n, m+n$. **94.** $1-z, 1+z$. **95.** $3m-4n, 3m+4n$. **96.** $x-1, x+1$. **97.** $c-3d, c+3d$. **98.** $2a-1, 2a+1$. **99.** $2r-3s, 2r+3s$. **100.** $1-3ab, 1+3ab$. **101.** $3x-8y, 3x+8y$. **102.** $1-9xy, 1+9xy$. **103.** $2-6rs, 2+6rs$. **104.** $3a-cd, 3a+cd$. **105.** $7xy-3, 7xy+3$. **106.** $1-5xy^2, 1+5xy^2$. **107.** $9a^2c-1, 9a^2c+1$. **108.** $3-ab, 3+ab$. **109.** $4-3, 4+3$. **110.** $\frac{1}{2}x-\frac{1}{3}y, \frac{1}{2}x+\frac{1}{3}y$. **111.** $5-3, 5+3$. **112.** $\frac{1}{3}-abc^2, \frac{1}{3}+abc^2$. **113.** $mn-\frac{1}{2}, mn+\frac{1}{2}$. **114.** $\frac{1}{3}a-\frac{1}{4}xy, \frac{1}{3}a+\frac{1}{4}xy$. **115.** $\frac{1}{4}m-\frac{1}{5}y, \frac{1}{4}m+\frac{1}{5}y$. **116.** $(a+x)-a, (a+x)+a$. **117.** $2-(b+c), 2+(b+c)$. **118.** $1-a^2, 1+a^2$. **119.** a^2-b^2, a^2+b^2. **120.** $x^2-y^2z^3, x^2+y^2z^3$.

In problems 91–120, the products of the binomial factors are the given quantities.

To find Two Unequal Binomial Factors of a Trinomial.—Page 118.

121. $a+2, a+1$. **122.** $b+4, b+3$. **123.** $y+5, y+4$. **124.** $a-2, a+1$. **125.** $a+2, a-1$. **126.** $x+3, x+2$. **127.** $x-3, x-2$. **128.** $x+3, x-2$. **129.** $x-3, x+2$. **130.** $x-6, x-3$. **131.** $x-2y, x-y$. **132.** $2b+3, 2b+2$. **133.** $2b-3, 2b-2$. **134.** $2b+3, 2b-2$. **135.** $2b-3, 2b+2$. **136.** $3a+3, 3a+2$. **137.** $3r-3, 3r-2$. **138.** $ax+4, ax-1$. **139.** $ax-4, ax+1$. **140.** a^2b^2-3, a^2b^2+2. **141.** $4s+3, 4s-2$. **142.** $4s-3, 4s+2$. **143.** $3a-5, 3a+3$. **144.** $3x+5, 3x-3$. **145.** $3y+5, 3y+4$. **146.** $3z+10, 3z+2$. **147.** $2b^2+4, 2b^2+3$. **148.** $2b^2-4, 2b^2-3$. **149.** $c-8d, c+2d$. **150.** $d+8e, d-2e$. **151.** $3xy+5, 3xy-4$. **152.** $3ab-5, 3ab+4$. **153.** $x+4a, x-3a$. **154.** $y-4a, y+3a$. **155.** $4a^2b-6, 4a^2b+1$. **156.** $4b^2c-3, 4b^2c+2$. **157.** $\frac{1}{2}a+6, \frac{1}{2}a+4$. **158.** $\frac{1}{3}m-6, \frac{1}{3}m+3$. **159.** $\frac{1}{4}y-7, \frac{1}{4}y-5$. **160.** $\frac{1}{4}z+7, \frac{1}{4}z-5$.

In problems 121–160, the products of the unequal binomial factors are the given quantities.

To find the Two Factors of the Sum or the Difference of Two Cubes.—Page 120.

161. $a+b, a^2-ab+b^2$. **162.** $x+z, x^2-xz+z^2$. **163.** $b-c, b^2+bc+c^2$. **164.** $c-y, c^2+cy+y^2$. **165.** $1+c, 1-c+c^2$. **166.** $x-1, x^2+x+1$. **167.** $r-s, r^2+rs+s^2$. **168.** $s+r, s^2-rs+r^2$. **169.** $1-ab, 1+ab+a^2b^2$. **170.** $xy+z, x^2y^2-xyz+z^2$. **171.** $ax+1, a^2x^2-ax+1$. **172.** $1-bc, 1+bc+b^2c^2$. **173.** $bc-x, b^2c^2+bcx+x^2$. **174.** $yz+ab, y^2z^2-abyz+a^2b^2$. **175.** $ab-2, a^2b^2+2ab+4$. **176.** $1+2an, 1-2an+4a^2n^2$. **177.** $rs-3, r^2s^2+3rs+9$. **178.** $y+mn, y^2-mny+m^2n^2$. **179.** $yz-2a, y^2z^2+2ayz+4a^2$. **180.** $ab+xy, a^2b^2-abxy+x^2y^2$. **181.** $2a-bc, 4a^2+2abc+b^2c^2$. **182.** $a+3x, a^2-3ax+9x^2$. **183.** $3-2bc, 9+6bc+4b^2c^2$. **184.** $xy+4a, x^2y^2-4axy+16a^2$. **185.** $5-az, 25+5az+a^2z^2$. **186.** $mn+4x, m^2n^2-4mnx+16x^2$. **187.** $6-de, 36+6de+d^2e^2$. **188.** $2xy-1, 4x^2y^2+2xy+1$. **189.** $\frac{1}{2}ab-4s, \frac{1}{4}a^2b^2+2abs+16s^2$. **190.** $3cd+\frac{1}{4}z, 9c^2d^2-\frac{3}{4}cdz+\frac{1}{16}z^2$.

In problems 161–190, the products of the factors found are the quantities given.

Common Factors.—Page 122.

1. H. c. f. $=a$; $1, ab$. **2.** H. c. f. $=a^2x^2$; x, a. **3.** H. c. f. $=b^2$; $1, 3b, 5$. **4.** H. c. f. $=2x$; $2xy, 4y^3, 3$.

5. $$\begin{array}{ll} \left.\begin{array}{l}5\\ 10bx^2\\ 15b^2x\end{array}\right\} & \begin{array}{r}=1\\ \div 5=2bx^2\\ =3b^2x\end{array} \\ \hline \text{H. c. f.} & =5\end{array}$$

6. $2x^2$; 4, $3xy^2$, $2y$. **7.** $5a^2c^2y$; $3c$, $4ay$. **8.** $2ab$; a^3, $4ab^2$, $3b$. **9.** $7x^2y^2z^3$; $3yz$, $5x$. **10.** $2x^2y^2$; 2, $4x^2y$, $3y^2$. **11.** $5d^2e^2$; e, $2ade^2$, 3. **12.** $7mx$; x^2, $2m^2x$, $3m^2y$. **13.** $6cr^2$; 3, ac^3r, $2cr^2$. **14.** $a^2d^2x^2$; d^2x, $2ax^2$, $3d$. **15.** $6c^2y^3$; 2, cy, $3c^2$. **16.** $2ab^2$; $2ab$, $3b^2(x-1)$, 4. **17.** $4x^2(a-b)$; 2, $3x$. **18.** $3c$; $2ac$, a^2+3c^2e-2cx. **19.** $2rs^2$; $rs-3r^2s^2+2$, $4r^3$. **20.** az^2; $3az$, $6-9a^2z^2+a$.

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21. $$\begin{array}{l} \left.\begin{array}{ll} a^2-b^2, & \text{or } (a+b)\ (a-b) \\ a^2+2ab+b^2, & \text{or } (a+b)\ (a+b)\end{array}\right\} \div a+b \begin{array}{l}=a-b\\ =a+b\end{array} \\ \hline \text{H. c. f.}=a+b\end{array}$$

22. $a-x$; $a-x$, $a+x$. **23.** $1-c$; $1+c$, $1-c$. **24.** $x+2$; $x+2$, $x-2$. **25.** $x-1$; y, $x+1$. **26.** $(x+y)\,(x)=x^2+xy$; 1, 4. **27.** $a-2a^2$; a, $2b$. **28.** $a+x$; $3a$, $a-x$. **29.** $x-1$; $x-1$, $x+1$. **30.** $a-1$; $a+1$, $a-1$. **31.** $x+1$; $x-1$, x^2-x+1. **32.** $1-x$; $1+x+x^2$, $1+x$. **33.** $x-1$; x^2+x+1, $x+1$. **34.** $3s^2(a+1)$; 1, $2(a-1)$. **35.** $m-n$; m^2+mn+n^2, $m+n$. **36.** x^2-y^2; 1, x^2+y^2. **37.** b^2-c^2; b^2+c^2, 1. **38.** $b-a$; $a(b^2+ab+a^2)$, $b+a$. **39.** $a-b$; 1, $a+b$, $a-b$. **40.** a^2-ab+b^2; $a+b$, 1. **41.** a^2+ax+x^2; 1, $a-x$. **42.** $a+b$; a^2-ab+b^2, $5c^2$. **43.** $x-3$; $x+3$, $x-3$. **44.** $x+y$; $x+y$, x^2-xy+y^2. **45.** $a+2$; $a-2$, $a+4$. **46.** $a+3$; $a-1$, $a-3$. **47.** $x-4$; $x+2$, $x+4$. **48.** $r-2$; $r+2$, $r+3$. **49.** $n+1$; $n-2$, $n-1$. **50.** $n+2$; 1, $n-1$.

Lowest Common Multiples.—Page 126.

1. $$\begin{array}{l} a;\ b,\ \text{quotient.} \\ b;\ a,\ \text{“} \\ \hline \text{L. c. m.}=ab\end{array}$$

2. L. c. m. $=6a^2b$; $3ab$, 1. **3.** L. c. m. $=10ax^3$; $2ax$, 1. **4.** L. c. m. $=12b^3y^3z$; $3y^2z$, $2b^2$. **5.** L. c. m. $=48a^3b^2c^3$; $4ab$, $3c^2$. **6.** $15x^3y^2z^4$; xy, z^3. **7.** $36a^2c^4e^2$; $2a$, $3ce$. **8.** $60x^3y^4$; $6x^3y^4$, $5x^2$, $4y^2$. **9.** $30c^3f^4s$; $10c^2f^3s$, $5f^2s$, $2c$. **10.** $60x^3y^2z^4$; $6yz^4$, $4xy^2z$, $3x^2$. **11.** $24a^2b^4x^3$; $24ax$, $2a^2b^4x^3$, $3b^3$. **12.** $abcd$; cd, ad, ab, bc. **13.** $60r^3s^4x$; $4r^2x$, $6r^3s^2$, $3s^2x$. **14.** $60a^3b^3x^4$; $20a^2b^3$, $15ax^4$, $12bx^3$. **15.** $140b^3c^5d$; $28b^3d$, $20b^2c^5d$. $10c^5d$, $7b^3c^3$. **16.** $72c^5d^4x$; $6d^3x$, $4c^2$, $3c^4d^2x$. **17.** $48a^4x^3y^2$; $6a^4x^3y^2$, $48a^2y^2$, $3xy^2$, $4a^3x^3$.

18. $30x^3y^5m^2$; $3x^2y^3m^2$, $6y^5$, $2x^3m$. **19.** $30a^2x^3+30a^3x^3$; $5a+5a^2$, $2x^2$. **20.** $6ax^2-6a$; $3x^2-3$, $6ax-6a$, $2ax^2-2a$, $6ax+6a$. **21.** x^3+x^2; $x+1$, x; h. c. f. $=x$. **22.** $6a^3-12a^2x$; $3a-6x$, $2a$; h. c. f. $=a$. **23.** $4b^3+4b^2$; $b+1$, $4b$; h. c. f. $=b$. **24.** $15xy^2-45xy$; $3xy-9x$, $15xy$, $5y-15$. **25.** a^2-x^2; $a-x$, 1; h. c. f. $=a+x$. **26.** x^4-y^4; 1, x^2+y^2; h. c. f. $=x^2-y^2$. **27.** a^2b+ab^2; b, a; h. c. f. $=a+b$. **28.** $abc(x-y)$; bc, a; h. c. f. $=x-y$. **29.** $a(a^2-1)$; $a-1$, a; h. c. f. $=a+1$. **30.** $b(b^2-1)$; b, $b+1$; h. c. f. $=b-1$. **31.** $ab(b^2-c^2)$; $(b-c)$, $(b+c)$; h. c. f. $=ab$. **32.** $4m^2(a^2-x^2)$; $2m(a-x)$, $(a+x)$; h. c. f. $=2m$. **33.** $b(b^2-x^2)$; $b-x$, b; h. c. f. $=b+x$. **34.** $a(1+x)^2$; a, $1+x$; h. c. f. $=1+x$. **35.** $a(a^2-1)$; a^2-1, a^2-a, a^2+a. **36.** $12(a-b)^2$; $3(a-b)$, 4; h. c. f. $=a-b$. **37.** $12(x^4-y^4)$; 3, $2(x^2-y^2)$; h. c. f. $=2(x^2+y^2)$. **38.** $10c(x^2-b^2)$; $5c(x+b)$, 2; h. c. f. $=x-b$. **39.** $a(1+x)^2$; $(1+x)$, a; h. c. f. $=1+x$. **40.** $10c(x^2-b^2)$; $5c(x+b)$, 2; h. c. f. $=x-b$. **41.** $12a^2b(x^2-y^2)$; $2b(x+y)$, $3a$; h. c. f. $=2a(x-y)$. **42.** $ab(9x^2-1)$; $b(3x+1)$, 1; h. c. f. $=a(3x-1)$. **43.** $4x^2(1-4a^2)$; 1, $2x(1+2a)$; h. c. f. $=2x(1-2a)$. **44.** a^2-b^2; $a-b$, $a+b$, 1. **45.** $1-x^2$; $1+x$, 1, $1-x$. **46.** a^3+b^3; a^2-ab+b^2, 1; h. c. f. $=a+b$. **47.** c^3-1; 1, c^2+c+1; h. c. f. $=c-1$. **48.** $(a+x)^2$; $a+x$, 1; h. c. f. $=a+x$. **49.** $(x-1)^2$; 1, $x-1$; h. c. f. $=x-1$. **50.** $1+x-x^2-x^3$; $1-x$, $1+x$; h. c. f. $=1+x$. **51.** $x(x^4+x^3-x-1)$; x^4+x^3-x-1, $x(x^2+x+1)$, $x(x+1)$. **52.** a^2-2a-3; $a-3$, 1; h. c. f. $=a+1$. **53.** $x^2-3x-10$; 1, $x-5$; h. c. f. $=x+2$. **54.** $a^3+3a^2-4a-12$; $a+3$, $a+2$; h. c. f. $=a-2$.

Fractions to Higher Terms.—Page 129.

1. $\frac{6}{8}$, $\frac{9}{12}$, $\frac{15}{20}$, $\frac{75}{100}$. **2.** $\frac{ab^2c}{2b^2c}$. **3.** $\frac{6y^2(a+x)}{6xy^3}$. **4.** $\frac{3ab(x-a)}{x^2-a^2}$.

5. $\frac{(1-x)(1-x)}{1-x^2}$. **6.** $\frac{(a+2)(a+2)}{a^2-a-6}$. **7.** $\frac{6x(a^2+ab+b^2)}{a^3-b^3}$.

8. $\frac{3(b-1)(1-b)}{3-3b^2}$, $\frac{3(b+1)}{3-3b^2}$.

Fractions to Lowest Terms.—Page 130.

1. $\frac{3}{4b}$. **2.** $\frac{2b}{3d}$. **3.** $\frac{2dx}{3c}$. **4.** $\frac{4z}{5x}$. **5.** $\frac{5rs}{6x^2y}$. **6.** $\frac{1}{2a-3b}$.

7. $\frac{b+c}{x}$. **8.** $\frac{1}{2}$. **9.** $\frac{b}{d}$. **10.** $\frac{c(c-2x)}{2ac^2(c-2x)}=\frac{1}{2ac}$. **11.** $\frac{1}{a+b}$.

12. $\frac{1}{a-b}$. **13.** $\frac{1}{a+x}$. **14.** $\frac{a}{b-1}$. **15.** $\frac{1}{r+s}$. **16.** $\frac{a-1}{4}$. **17.** $\frac{x+y}{2}$.

18. $\frac{x+y}{x-y}$. **19.** $\frac{ay}{a+y}$. **20.** $\frac{c+d}{2(c-d)}$. **21.** $\frac{1}{a+b}$. **22.** $\frac{x}{x-y}$.

23. $\frac{c}{a-2c}$. **24.** $\frac{x+y}{x^2+xy+y^2}$. **25.** $\frac{1}{b+1}$. **26.** $\frac{1}{3(x-y)}$. **27.** $\frac{a-1}{2b}$.

28. $\frac{1}{x^2-1}$. **29.** $\frac{a+x}{a-x}$. **30.** $\frac{x^2-xy+y^2}{x-y}$. **31.** $\frac{1}{3a+2y}$.

32. $\frac{a+y}{a^2+ay+y^2}$. **33.** $\frac{s-3}{s^2-2s+4}$. **34.** $\frac{x+2}{x+3}$. **35.** $\frac{r-3}{r-5}$. **36.** $\frac{x-1}{x-2}$.

Fractions to Equivalent Fractions having the Lowest Common Denominator.—Page 132.

1. $\frac{ac}{bc}, \frac{b^2}{bc}$. **2.** $\frac{ayz}{xyz}, \frac{bxz}{xyz}, \frac{cxy}{xyz}$. **3.** $\frac{drx}{cdr}, \frac{cry}{cdr}, \frac{cdz}{cdr}$.

4. $\frac{a^2x^3,\ 2b^2x^2,\ a^3b}{ax^2y}$. **5.** $\frac{2adx,\ 3abx,\ 4c}{a^2d^2x^2}$. **6.** $\frac{b^2c^3,\ b^2d^3,\ ac^2d^2}{b^2c^2d^2}$.

7. $\frac{m^2r^2, n^3r^2, m^5n}{mnr^2}$. **8.** $\frac{2bxz, x^2y^2, 3cdyz}{x^2y^2z}$ **9.** $\frac{c^2, ad, rx}{acx}$. **10.** $\frac{a^2x, d^2x, xy^2}{ady}$.

11. $\frac{8c^3, 9ad, 5ac}{ac^2d}$. **12.** $\frac{a^2bc, ab, a^3b}{a^2b^2}$. **13.** $\frac{16ayz,\ 18bxz,\ 12sy^2}{24xy^2z}$.

14. $\frac{abcm, acdr, b^2x}{ab^2c^2d}$. **15.** $\frac{8abxy,\ 9acx^2, 12d^2y}{12ax^2y}$. **16.** $\frac{4b, 3ax-3bx}{a^2-b^2}$.

17. $\frac{ab - 2a^2b,\ 3}{1 - 4a^2}$. **18.** $\frac{3,\ (3 - x)^2}{9 - x^2}$. **19.** $\frac{x - y, 2x^2 - 2xy, 2y}{2x - 2y}$.

20. $\frac{3a^2 + 3ab,\ a - b,\ 18a}{6a}$. **21.** $\frac{3xy + 3y^2,\ \ 4x^2 - 4xy,\ \ 2acxy}{12xy}$.

22. $\frac{4a^2(1 + a),\ 2a,\ ab(1 - a^2)}{2a(1 - a^2)}$. **23.** $\frac{2ac(x - 1),\ 4xy,\ \ y(x + 1)}{x^2 - 1}$.

24. $\frac{a(a-x),\ b(a+x),\ c}{a^2-x^2}$. **25.** $\frac{2(a^2-9), 3a^2(a-3), 5a(a+3)}{a(a^2-9)}$.

26. $\frac{3x(a+b)^2,\ 12x^2(a-b),\ 2a(a^2-b^2)}{3x(a^2-b^2)}$. **27.** $\frac{3(1+2a),\ x(1-2a),\ 1-2a}{1-4a^2}$.

28. $\frac{3a(1 + 2x),\ 2(1 - 4x^2),\ 2a}{1-4x^2}$. **29.** $\frac{x(b + a),\ ax,\ 2a(b^2 - a^2)}{b^2-a^2}$.

30. $\frac{3a(x-y),\ 4b(x+y),\ 3(x^2-y^2)}{x^2-y^2}$.

NOTE.—These results may be arranged by beginners with the least common denominator placed under each numerator, as per examples 1, 2, 3, above.

To Clear Equations of Fractions.—Page 135.

1.

$$\begin{array}{ll} & \frac{x}{3}+\frac{x}{2}=5 \\ & \underline{\ \ 2+3=5\ \ } \\ \text{Clearing of fractions,} & 2x+3x=30 \\ \text{Collecting,} & 5x=30 \\ \text{And} & x=6 \end{array}$$

Substitute value of x, and verify equations, in examples 1–25.

2. 30. **3.** 12. **4.** 6. **5.** 6. **6.** 6. **7.** 2. **8.** 90. **9.** 12.
10. 12. **11.** 6. **12.** 8. **13.** 12. **14.** 8. **15.** 6. **16.** 6.
17. 10. **18.** 5. **19.** 1. **20.** 3. **21.** 5. **22.** 3. **23.** 12.
24. 6. **25.** 7. **26.** 4. **27.** $2\frac{1}{2}$. **28.** 1. **29.** 5. **30.** 33.
31. 1. **32.** 12. **33.** 4. **34.** 3. **35.** $2\frac{1}{2}$. **36.** 12. **37.** 10.
38. 5. **39.** 1. **40.** 5. **41.** $\frac{1}{2}$. **42.** 8. **43.** 7. **44.** 6. **45.** 1.
46. 2. **47.** $\frac{a^2}{2c}$. **48.** $a+b+c$. **49.** $\frac{abc}{bc+ac+ab}$. **50.** $2a$.

Review Examples and Problems.—Page 137.

1. $15ab^2-a^2x^4$. **2.** $17c^2d-m^4r^3$. **3.** $11x^2y^3-v^2x^3$. **4.** $-2a^4b^3-d^3e^2+4c^2d^4$. **5.** $20s^2t^4+u^3v^2$. **6.** $5\frac{1}{3}a^2x-3\frac{7}{8}b^2y^3+1\frac{5}{8}c^4z^3$. **7.** $c^3d^4+3\frac{1}{4}a^4x^2$. **8.** $\frac{7}{8}b^3c^2+\frac{1}{8}cd^3+2\frac{1}{4}a^2y^3$. **9.** $4bc^2-9d^2e^3+4c^3f^4+7a^5x^3$. **10.** $6a^2x-6ax^3-7c^3z^4$. **11.** $4b^3x^2-8d^2z^4+5e^3n^5$. **12.** $\frac{1}{4}c^2w^3+3\frac{3}{4}d^3x^4+4a^4z^3$. **13.** $1\frac{1}{3}a^3x^2y+\frac{1}{2}x^4y^3+\frac{2}{3}y^2z^4$. **14.** $-3v^4w^3-1\frac{1}{3}t^3u^2+7u^2v^4$. **15.** $12ab^2+b^2c+c^2d^3$; $4b^2c-c^2d^3+a^4x$. **16.** $-b^2c^3-7c^2x^4+9d^3y^5$; $5b^2c^3-c^2x^4+12d^3y^5$. **17.** $\frac{3}{4}a^3x^2+\frac{3}{4}c^4z^3-\frac{3}{8}a^3y^4$; $-\frac{1}{4}a^3x^2+1\frac{1}{4}b^2y^4+\frac{3}{4}c^4z^3$. **18.** $-\frac{2}{3}a^3x^2-2\frac{1}{3}c^2z^3-d^4s^3$; $-\frac{1}{3}a^3x^2-4d^4s^3-4b^3y^4$. **19.** $\frac{1}{2}bx^2+3c^3y^2+3d^3z^4$; $7c^3y^2-d^3z^4$. **20.** $20-14-6=0$. **21.** $20-14+6=12$. **22.** $20+2-6=16$. **23.** $20+10-\overline{4+2}=24$. **24.** $30+9=39$. **25.** $10x-8y$. **26.** $10x-8y$. **27.** $x+2y$. **28.** $8y-7x-6y+8x=x+2y$. **29.** $2x-2y+2x+2y-2x=2x$. **30.** $10-8-6-3+3=-4$. **31.** $\{40-10-10\}-[3+1]=16$. **32.** $12-[8-13-9]-(3+1)=22$. **33.** $(1-2a)-(-1-3a)-4a=2-3a$. **34.** $(4a-2b)-(2b-2a)+a=7a-4b$. **35.** $(6x-2y)-(3x-2y)-3y=3x-3y$. **36.** $8a^2-18b^2x^2$; $10a^2-4abx$; $-8abx-3a^2x^2$. **37.** $24a^2c^2-6b^4$; $24a^2c^2-12b^4+ab^2c$; $-a^2c^2-ab^2c$. **38.** $12c^2x^2-14cxy^2-10y^4$; $3c^2x^2-14cxy^2$; $6c^2x^2-14cxy^2+20y^4$. **39.** $2ab^2-c^3d^2$; $-ab^2+7bc$; $-2a^2b^4+17ab^3c-21b^2c^2$. **40.** $2ab^2-2cd^2$; $-cd^2+2d^3e^2$; $-3c^2d^4+9cd^5e^2-6d^6e^4$. **41.** $-bx^2+4c^4y^3$; $-6bx^2-3d^2z$; $24b^2x^4-6d^4z^2$. **42.** $25a^2+70ab+49b^2$. **43.** $36x^2-96xy+64y^2$. **44.** $16a^2b^2+40abc+25c^2$. **45.** $9b^2-24abx+16a^2x^2$. **46.** $4x^4+20x^2y^2+25y^4$. **47.** $36a^2-25b^2$. **48.** $16x^2-36y^2$. **49.** $9a^2b^2-16y^2$.

50. $25x^2-49y^2z^2$. **51.** $16a^4b^2-25c^4$. **52.** $25a^2-60a+35$. **53.** $16b^2+36b+18$. **54.** $9a^2x^2-3ax-20$. **55.** $36-12a-15a^2$. **56.** $25a^4+10a^2-48$. **57.** $16a^2b^4-8ab^2-35$. **58.** $25-10xy-8x^2y^2$. **59.** a^4-b^4. **60.** x^4-2x^2+1. **61.** x^3+y^3.

Solve problems 42–61 by actual multiplication (same results).

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62. $4a-6bx$; $6a-12bx+3cy$; $-8bx+4cy$. **63.** $3b^2+6ac$; $-b^2-5ax^2$; $-2b^2-2ax^2-z^4$. **64.** $-a^2c^4-5b^4d^2+10c^2e^2$; $4a^2c^4-9c^2e^2$; $2ac^2+3ce$. **65.** $16b^2-36a^2x^2$; $4b-6ax$; $-5b+7z$. **66.** $-5s^2t^3-3y^4z^4$; $4a^2b^2-9y^4z^4$; $2ab+3y^2z^2$. **67.** $3a^2b^3-5x^3y^2$; $9a^4b^6-25x^6y^4$; $+\frac{1}{4}x^6y^4-\frac{2}{3}a^2x^2$. **68.** $a-b$. **69.** $2b+y$. **70.** $2x-2y$. **71.** $3ax+4b$. **72.** $4a^2-3ab$. **73.** $5a^2b+4c^2$. **74.** $2x+5$. **75.** $3ab+9$. **76.** $5a^2x-10$. **77.** $4bc^2-5$. **78.** $6a^2y^2-8$. **79.** $7bx^3+9$. **80.** $3c-1$. **81.** $4x^2-3y$. **82.** $4a^2-2a+1$. **83.** $9x^2y^2+6xyz+4z^2$. **84.** $3a^2b^2+5x^3y^2$. **85.** $4a^2-8a^3b^2+16a^4b^4$.

Solve problems 68–85 by actual division (same results).

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86. $1-2a+\frac{8a^2}{1+2a}$. **87.** $2a-3b+\frac{18b^2}{2a+3b}$. **88.** $x^2+xy+y^2+\frac{2y^3}{x-y}$. **89.** $5a^2b-4c^2+\frac{32c^4}{5a^2b+4c^2}$. **90.** $4a^2b^2-6abxy+9xy-\frac{54x^3y^3}{2ab+3xy}$. **91.** $9b^4c^2+12ab^2cx^2+16a^2x^4+\frac{128a^3x^6}{3b^2c-4ax^2}$. **92.** $a-3x+\frac{4x^2}{a+x}$. **93.** $2b^2+6c^2+\frac{16c^4}{2b^2-2c^2}$. **94.** $3a^2x-12y^2+\frac{64y^4}{3a^2x+4y^2}$.

The required operations in problems 86–94 give the original dividends.

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95. $b-a$, $b-a$. **96.** $ab+2c$, $ab+2c$. **97.** $2a-3x$, $2a-3x$. **98.** $3b^2+4c^2$, $3b^2+4c^2$. **99.** $4ac-5x$, $4ac-5x$. **100.** $5c^2+6ay^2$, $5c^2+6ay^2$. **101.** $6a^2x^2-5x^2y^2$, $6a^2x^2-5x^2y^2$. **102.** $x+5$, $x-7$. **103.** $ay+6$, $ay-4$. **104.** $2x+8$, $2x-5$. **105.** $3ab-8$, $3ab+6$. **106.** $4bc-10$, $4bc+4$. **107.** $5a^2b^3+10$, $5a^2b^3+5$. **108.** $6b^3y^2-7$, $6b^3y^2-6$. **109.** $2a^2-3x^2$, $2a^2+3x^2$. **110.** $3ac-4b$, $3ac+4b$. **111.** $5bc^2-6ax^2$, $5bc^2+6ax^2$. **112.** $6c^3x^2-5ay^2$, $6c^3x^2+5ay^2$. **113.** a^2+3b, $a^4-3a^2b+9b^2$. **114.** $2ab+2$, $4a^2b^2-4ab+4$. **115.** $3bx^2-4a$, $9b^2x^4+12abx^2+16a^2$. **116.** $4a^2y^2+5$, $16a^4y^4-20a^2y^2+25$. **117.** $a+1$, $2a+5$. **118.** $4x-1$, $x-2$. **119.** $3b+4$, $2b+1$. **120.** $5y-4$, $3y+2$.

The products of the factors found will be the given composite quantities.

121. H. c. f. $=6b^2c^3d$; $2a^2bc$, $3de$. **122.** H. c. f. $=5ax^2$; 3, $5ac^4x^3$, $2a^2b^2x$. **123.** $2a(c-1)$; b, $3c$. **124.** $3bc^2(b^2+2c)$; bc, 3. **125.** $b(a-1)$; 1, $2b(a+1)$. **126.** $1-x$; $1+x$, $1-x$. **127.** $2(a-x)$; $2b$, $3c^2(a+x)$. **128.** $5x-4$; $3x$, $a^2(5x+4)$. **129.** $x-y$; $x+y$, $x-y$. **130.** $2x+3y$; $2x-3y$, $2x$. **131.** $2ab(a^2-3b)$; $3a$, $4b^2$. **132.** $3a(2x-3z)$; 1, $2a(2x-3z)$. **133.** $a(2a-3)$; 1, $6(2a+5)$. **134.** $2b(x+4)$; $x-6$, $2b(x-4)$. **135.** $a+x$; 1, $a-x$, a^2-ax+x^2. **136.** $x-1$; 1, $x-1$, x^2+x+1. **137.** $3a+2$; a, $3a+2$, $3a-5$. **138.** $2(1-x)$; $2(1-x)$, $3(1+x)$, $(2-x)$. **139.** $a(3a-2b)$; $2a$, $6b(3a+2b)$, $3b^2(9a^2+6ab+4b^2)$. **140.** $30a^2b^3x^4y^4$; $15ab^3xy^3$, $10a^2by^2$, $6x^4$; h. c. f. $=y$. **141.** $24b^3c^3y^4z^5$; $6bc^3y^2$, $4b^3z^2$, $3cy^4z^4$; h. c. f. $=2z$. **142.** $30a^3c^2d^2x^4y^5$; $6a^3cxy^5$, $2c^2d^2y$, $3a^3dx^4$; h. c. f. $=5$. **143.** $12a^3b^4c^2(a+x)$; $3a^2b^2c(a+x)$, $2b^4(a+x)$, $12a^2c^2$; h. c. f. $=a$. **144.** $6x^2y^3(a^2-1)$; $2x(a-1)$, y^2; h. c. f. $=3xy(a+1)$. **145.** $10ab(1-x)$; $2b$, 1; h. c. f. $=5a(1-x)$. **146.** $6x^2(a^2-1)$; $2x^2$, $a+1$; h. c. f. $=3(a-1)$. **147.** a^2-b^2; $a+b$, 1; h. c. f. $=a-b$. **148.** $(1-x)^2$; $1-x$, $1+x$, 1; h. c. f. $=$ *prime.* **149.** $24xy^2(a^3+a^2x-ax^2-x^3)$; $3x(a-x)$, $4y^2(a+x)$; h. c. f. $=2(a+x)$. **150.** $12a^3x^2(x^2-y^2)$; $2a^2(x+y)$, $3x^2$; h. c. f. $=2a(x-y)$. **151.** $24abc(a^2-c^2)$; $3(a+c)$, $2bc$; h. c. f. $=4a(a-c)$. **152.** $12ab^2(1-x-x^2+x^3)$; $2a(1+x)$, $3b^2(1-x)$; h. c. f. $=2(1-x)$. **153.** $8x^3-12x^2y-18x^2y+27y^3$; $2x-3y$, $2x+3y$; h. c. f. $=2x-3y$. **154.** $12ab^3c^2(x^2-y^2)$; $6b^3c^2(x-y)$, $3b^3(x+y)$, $2ac$; h. c. f. $=2$. **155.** $18a^3(1-x^2)$; $6(1+x)$, $3a(1-x)$, $2a^2$; h. c. f. $=3a$. **156.** x^2-y^2; $x+y$, 1, 1; h. c. f. $=x-y$. **157.** $48a^2bc(a^2-c^2)$; $6ab(a+c)$, $4abc$, $3c(a-c)$; h. c. f. $=4a$. **158.** $b^3-6b^2-16b+96$; $(b-4)(b-6)$, $b-6$, $b-4$; h. c. f. $=b+4$. **159.** $8a^3-16a^2-70a+150$; $(2a-5)(2a+6)$, $2a+6$, $2a-5$; h. c. f. $=2a-5$. **160.** $1-x+x^3-x^4$; $(1-x)(1-x+x^2)$, $1-x+x^2$, $1-x$; h. c. f. $=1+x$.

161.
$$\begin{aligned}5x-15+6x-10-7x+12&=3x-8\\ \underline{25-15+30-10-35+12}&\underline{=15-8}\\ 11x-10x&=-20+25\\ x&=5\end{aligned}$$

In problems 161–187, substitute the values of x, and verify the equations.

162. 2.
163. 6.
164. 4.
165. 1.
166. 2.
167. 2.
168. 8.
169. 12.
170. 8.
171. 24.
172. 5.
173. 34.
174. 3.
175. 1.
176. 12.
177. 6.
178. 8.
179. 5.
180. 21.
181. 9.
182. 19.
183. 1.
184. 8.
185. 6.
186. 8.
187. 18.

188. 7. **189.** 30 cents, Martha; 20 cents, Ruth; 10 cents, Mary. **190.** 18 greater, 15 less. **191.** 16 rabbits, Harry; 30 rabbits, Frank. **192.** 60 cents, John; 40 cents, James. **193.** 15 cents, Edith; 35 cents, Fanny; 50 cents, Ida. **194.** 90 bu. **195.** 65 Java, 55 Mocha. **196.** 42 gal. **197.** 40 yr., father; 10 yr., son. **198.** $20,000.

199. Let $x =$ number acres,

Then $\frac{3x}{5} - 10$ acres $= \frac{5x}{8} - 12$ acres.

$$24x - 400 = 25x - 480$$

$$x = 80$$

Therefore, he had 80 acres at first.

200. 20 yr. **201.** 2 days. **202.** 3 hrs. **203.** 18 marbles, Arthur; 24 marbles, Walter. **204.** $1200, A; $1600, B. **205.** 36 ft., length; 24 ft., width. **206.** 12 in., length; 9 in., width.

207. Let $x =$ cost of first,

Then $x + \frac{x}{4} = \$6$

$$\frac{5x}{4} = 6$$

$$x = 4.80$$

Let $x =$ cost of second,

Then $x - \frac{x}{4} = \$6$

$$\frac{3x}{4} = 6$$

$$x = 8$$

Therefore, the first cost $4.80, and the second $8.

208. 6 days. **209.** 6 days.

210. Let $x =$ salary,

Then $\frac{x}{5} =$ amount A saved,

And $\frac{x}{5} - \$100 =$ amount B saved.

Hence, $x - 500 = 500$

$$x = 1000$$

Therefore, the salary of each was $1000.

211. 60 cents. **212.** $12,000.

213. Let $x =$ amount B put in,

And $\frac{2x}{3} =$ amount A put in,

Then $\frac{x}{5} =$ A bought of B,

Hence, $\frac{4x}{5} + \frac{2x}{3} + \frac{x}{5} = \$3200$

$$\begin{aligned} 12x + 10x + 3x &= 48000 \\ 25x &= 48000 \\ x &= 1920 \\ \frac{2x}{3} + \frac{x}{5} &= 1664 \\ \frac{4x}{5} &= 1536 \end{aligned}$$

Therefore, A then had \$1664, and B had \$1536.

214. \$30, A; \$40, B; \$50, C.

215. 25 cents, first; 40 cents, second; 55 cents, third.

216. \$10, first; \$12, second; \$18, third.

217. \$200, B; \$400, A. **218.** 20 miles.

219. Let $x =$ number of plums,

Then $\frac{2x}{5} =$ cost of all,

And $\frac{x}{6} + \frac{2x}{5} - 5 = \frac{2x}{5}$

$$\begin{aligned} \frac{x}{6} &= 5 \\ x &= 30 \end{aligned}$$

Therefore, Ned bought 30 plums.

220. Let $x =$ money,

Then $\frac{3x}{4} + 1 =$ money after earning \$1,

And $\frac{9x}{16} + \frac{3}{4} = \$3$

$$\begin{aligned} 9x + 12 &= 48 \\ 9x &= 36 \\ x &= 4 \end{aligned}$$

Therefore, Frank had \$4 at first.

221. $2\frac{2}{11}$ days.

222. Let $x =$ number of days,

Then $\underline{\frac{1}{4}-\left(\frac{1}{10}+\frac{1}{15}\right)=\frac{1}{x}}$

$\frac{1}{4}-\frac{1}{6}=\frac{1}{x}$

$\frac{1}{12}=\frac{1}{x}$

$x=12$

Therefore, C can do it in 12 days.

223. Let $x =$ time past midnight,

Then $\underline{\frac{x}{3}=12-x}$

$x=36-3x$

$x=9$

Therefore, it is 9 o'clock A. M.

224. $7\frac{1}{5}$, or 12 min. past 7 P. M. **225.** \$5, Henry; \$25, George.

226. Let $x =$ rate of speed,

Then $\frac{100}{x}=$ time,

And $\underline{(x+5)\frac{100}{x}=120 \text{ miles},}$

$100x+500=120x$

$-20x=-500$

$x=25$

Therefore, the train runs 25 miles an hour.

227. Let $x =$ perpendicular,

Then $x-3=$ base,

And $\underline{(x-3)+6=}$ hypothenuse.

$x=12$ in., perpendicular,

$x-3=9$ in., base,

$x+3=15$ in., hypothenuse.

228. Let $x =$ amount at first,

Then $2x-10$ cents $=$ amount after spending first time,

And $4x-20$ cents -10 cents $=$ amount after spending second time.

Hence, $\underline{4x-30 \text{ cents}=10 \text{ cents}.}$

$4x=40$

$x=10$

Therefore, John had 10 cents at first.

229. 19 pies. **230.** 20 days.

231. Let $x =$ number of days absent,
And $20 \text{ days} - x =$ number of days present,
Then $1x =$ sum forfeited,
And $3(20 - x) =$ sum earned,
Hence, $\underline{\$60 - 3x - x = \$50.}$
$-4x = -10$
$x = 2\frac{1}{2}$
Therefore, he was absent $2\frac{1}{2}$ days.

232. $s - (a + b)$, 5; $s = 12$, $a = 3$, $b = 4$. **233.** $\frac{s+d}{2}$, larger, $\frac{s-d}{2}$, smaller, 7, 5; $s = 12$, $d = 2$. **234.** $\frac{p}{ab}$, 4; $p = 24$, $a = 2$, $b = 3$. **235.** $d - ab$. **236.** $ab + c$. **237.** $\frac{a}{b}$, or $a \div b$. **238.** $a \div bc$, or $\frac{a}{bc}$. **239.** $c + \frac{cr}{100}$. **240.** $s \div \left(1 + \frac{r}{100}\right)$. **241.** prt. **242.** $i \div rt$. **243.** $i \div rp$. **244.** $i \div pt$. **245.** $p(1 + rt)$. **246.** $a \div (1 + rt)$.

In problems 235–246, give literal values, and find the numerical results.

247. Let $x =$ first part,
And $a - x =$ second part,
Then $\frac{x}{m} = a - x$.
Or $x = ma - mx$,
And $x + mx = ma$,
Hence, $x = \frac{ma}{1 + m} = \frac{16 \times 3}{1 + 3} = 12$, first part,
$a - x = a - \frac{ma}{1 + m} = \frac{a}{1 + m} = \frac{16}{1 + 3} = 4$, second part.
$a = 16$, $m = 3$

248. $\frac{amn}{n - m}$; $m = 2$, $n = 3$, $a = 2$. **249.** $\frac{ab}{b + a}$; $a = 12$, $b = 4$.

250. $\frac{an}{a - n}$; $n = 3$, $a = 12$. **251.** $\frac{abc}{bc + ac + ab}$; $a = 4$, $b = 6$, $c = 12$.

252. $\frac{a}{c}$ = part first pays; $\frac{b}{c}$ = part second pays; $a = 6$, $b = 8$, $c = 14$.

253. $\frac{12n}{1 + n}$; $n = 3$, $x = 9$. **254.** $\frac{abn}{b + a}$; $a = 3$, $b = 9$, $n = 8$.

255. $\frac{br + c}{a + b}$; $r = 20$, $b = 1$, $a = 3$, $c = 50$. **256.** $\frac{ax + by + cz}{a + b + c}$.

Find the numerical values in problems 248–256.

Reduction of Fractions.—Page 152.—To Change a Fraction to an Entire or a Mixed Quantity.

1. $2\frac{1}{2}a$. **2.** $2\frac{1}{3}a$. **3.** $2\frac{1}{4}c$. **4.** $4ax$. **5.** $1+\frac{d}{a}$. **6.** $b+\frac{c^2}{b}$.

7. $1-\frac{1}{4x}$. **8.** $2x-\frac{2}{5x}$. **9.** $1+\frac{y}{x}$. **10.** $1-2d$. **11.** $2b-\frac{b^2}{a+b}$.

12. $3x+2y+\frac{2y^2}{x-y}$. **13.** $a+b$. **14.** $c-\frac{3c^3}{b+c^2}$. **15.** $a-b+\frac{2b^2}{a+b}$.

16. $a+b$. **17.** $b-2+\frac{3}{b}$. **18.** $3b+1-\frac{1}{4b}$. **19.** $3x-4y+\frac{4y^2}{x+y}$.

20. $x+1+\frac{2}{x-1}$. **21.** $a-2$. **22.** a^2+ab+b^2. **23.** $4x^2-6xy+$
$9y^2-\frac{54y^3}{2x+3y}$. **24.** $ab - x^2y^2 + \frac{2x^4y^4}{ab+x^2y^2}$. **25.** $a^4b^4 + a^2b^2x^2y^2 +$
x^4y^4. **26.** $b^3+2bc^2+\frac{2bc^4}{b^2-c^2}$. **27.** $2x+y+\frac{y^2}{x-y}$. **28.** $3c-2-\frac{1}{cd}$.
29. $2a+3z$.

Performing the required operations with the results (examples 1–29) will give the original fractions.

To Change a Mixed Quantity to the Form of a Fraction.—Page 154.

1. $\frac{2a+b}{2}$. **2.** $\frac{a(3b+1)}{3}$. **3.** $\frac{a(8-b)}{4}$. **4.** $\frac{12b+2c}{3}$. **5.** $\frac{20c-3y}{4}$.

6. $\frac{30ax+y}{5}$. **7.** $\frac{y(30y-z)}{10}$. **8.** $\frac{7cx+y-z}{c}$. **9.** $\frac{8a^3-b-c}{2a}$.

10. $\frac{2b}{b+c^2}$; $2-\frac{2c^2}{b+c^2}$. **11.** $\frac{2c^2}{b+c^2}$; $2-\frac{2b}{b+c^2}$. **12.** $\frac{ax-a+1}{a}$.

13. $\frac{12a^2b-b^2-4}{4a}$. **14.** $\frac{4a-4ab^2-c}{4a}$. **15.** $\frac{x+y+1}{y+1}$. **16.** $\frac{7a+5c^2}{7}$;
$a+\frac{5c^2}{7}$. **17.** $\frac{9a^2+b}{4a}$; $2a+\frac{a^2+b}{4a}$. **18.** $\frac{a^2-3ab}{2a-3b}$; $a-\frac{a^2}{2a-3b}$.

19. $\frac{1+ax+a}{a}$ **20.** $\frac{8a+b}{2}$; $4a+\frac{b}{2}$. **21.** $\frac{b^2+c^2}{b+c}$. **22.** $\frac{a^2}{1-a}$. **23.**
$\frac{x^2+x-2}{x+3}$. **24.** $\frac{8-a^2+a}{3+a}$; $-a+4-\frac{4}{a+3}$. **25.** $\frac{5-a^2+a}{2-a}$; $a+1+\frac{3}{2-a}$.

26. $\frac{x^2-4x+4}{2x}$; $-2+\frac{x}{2}+\frac{2}{x}$.

27. $\frac{2x^2}{x-y}$; $2x+2y+\frac{2y^2}{x-y}$.

28. $\frac{4b^2-9c^2-2bc}{2b-3c}$; $2b+2c-\frac{3c^2}{2b-3c}$.

Performing the required operations (examples 1–28) will give the original mixed quantities, except in examples 11, 16, 17, 18, 20, 24, 25, 26, 27, 28.

Addition and Subtraction of Fractions.—Page 156.

1. $\frac{a+b}{2}$; $\frac{a-b}{2}$.

2. $\frac{5c-5b}{bc}$; $\frac{5c+5b}{bc}$.

3. $\frac{ad+ac}{cd}$; $\frac{ad-ac}{cd}$.

4. $\frac{b^2-5c}{5b}$; $\frac{b^2+5c}{5b}$.

5. $\frac{25+bc}{5b}$; $\frac{25-bc}{5b}$.

6. $\frac{2a-1}{6x}$; $\frac{2a+1}{6x}$.

7. $\frac{3bx+6a^2x}{ab}$; $\frac{3bx-6a^2x}{ab}$.

8. $\frac{8c-bc}{4ab^2}$; $\frac{8c+bc}{4ab^2}$.

9. $\frac{10bd-3a^2}{4a^2cd}$; $\frac{10bd+3a^2}{4a^2cd}$.

10. $\frac{a+7b}{12}$; $\frac{7a+b}{12}$.

11. $\frac{7a-b}{a^2-b^2}$; $\frac{a-7b}{a^2-b^2}$.

12. $\frac{45cx^2+14x}{35abc}$; $\frac{45cx^2-14x}{35abc}$.

13. $\frac{1}{a}$; $\frac{3a-1}{a(a+1)}$.

14. $\frac{b}{a+b}$; $\frac{2a-b}{a+b}$.

15. $\frac{9c+d}{c^2-d^2}$; $-\frac{c+9d}{c^2-d^2}$.

16. $\frac{c+9d}{20}$; $\frac{9c+d}{20}$.

17. $\frac{7a^2-b^2}{6(a-b)}$; $-\frac{5a^2+b^2}{6(a-b)}$.

18. $\frac{4b}{b^2-1}$; $\frac{2(b^2+1)}{b^2-1}$.

19. $\frac{6x}{x^2-4}$; $\frac{4+2x^2}{x^2-4}$.

20. $-\frac{2a^2+22}{2a+15-a^2}$; $\frac{14a+2}{2a+15-a^2}$.

21. $\frac{2x^2+8y^2}{x^2-4y^2}$; $-\frac{8xy}{x^2-4y^2}$.

22. $\frac{4a^2}{1-a^4}$; $\frac{2+2a^4}{1-a^4}$.

23. $\frac{2b(2b-c)}{b^2-c^2}$; $\frac{2bc}{b^2-c^2}$.

24. $\frac{2x-2x^2-2xy-2y^2}{x^3-y^3}$; $\frac{2x+2x^2+2xy+2y^2}{x^3-y^3}$.

25. $\frac{2(a^2+ab+b^2)}{a^3+a^2b+ab^2+b^3}$; $\frac{2ab}{a^3+a^2b+ab^2+b^3}$.

26. $-2ab$; $2(a^2+b^2)$.

27. $\frac{235a+27x}{45}$; $\frac{215a-63x}{45}$.

28. $\frac{2(yz+xz-xy)}{xyz}$; $\frac{2(yz-xz+xy)}{xyz}$.

29. $\frac{7x+y}{18}$; $\frac{5x+11y}{18}$.

30. $\frac{6x}{x^2-y^2}$; $\frac{-6y}{x^2-y^2}$.

The result of the required operation in each example is given first; the result of the reversed operation follows.

Multiplication of Fractions.—Page 158.

1. $\frac{4x}{y^2}$. **2.** $-\frac{ac}{bd}$. **3.** $\frac{x}{y}$. **4.** $\frac{15ay}{x}$. **5.** $\frac{a^2}{10b}$. **6.** $\frac{4ay}{3x}$.

7. $\frac{3ay^2}{4bx}$. **8.** $\frac{4ax^2}{3by}$. **9.** $\frac{4bc}{9a}$. **10.** $\frac{6x^2y}{a^2-b^2}$. **11.** $\frac{2a}{x+3}$.

12. $\frac{c-d}{3(c+d)}$. **13.** b^2-4b+4. **14.** $\frac{a-1}{x+3}$. **15.** 1. **16.** $\frac{a^2(a-c)}{2bx^2}$.

17. $\frac{1}{4(x-5)}$. **18.** $\frac{3}{2(a+4)}$. **19.** $\frac{xy}{1-x}$. **20.** $\frac{1}{a}$. **21.** a^2-c^2.

22. $\frac{x^2-xy+y^2}{x^2+xy+y^2}$. **23.** $\frac{1-x}{2ac+1}$. **24.** $\frac{a+b}{a+3b}$. **25.** $\frac{3}{x+1}$.

26. $\frac{1}{a^2+ab+b^2}$. **27.** $\frac{ax}{(a-b)(x-y)}$. **28.** $-\frac{1}{(x-2)^2}$.

29. $\frac{a(x+y)}{a^2-4}$. **30.** $\frac{x^3}{x-3}$; $\frac{x^3-a^3}{x-3}=x^2+3x+9+\frac{27-a^3}{x-3}$.

Division of Fractions.—Page 161.

1. $\frac{1}{a^2}$. **2.** a^3. **3.** $\frac{ac}{b^2}$. **4.** $-\frac{ad}{cx}$. **5.** $\frac{1}{18cx}$. **6.** $\frac{x}{15ay^3}$.

7. $\frac{4ab}{y^2}$. **8.** $\frac{12b^2c}{x}$. **9.** $\frac{y}{ax^2}$. **10.** $\frac{9ac}{10x^2y}$. **11.** $\frac{y}{x^2}$. **12.** a.

13. $\frac{6x^2y}{a^2-b^2}$. **14.** $\frac{3b}{2a(x-y)}$. **15.** $(a+2)^2$. **16.** $\frac{a-x}{1+a}$. **17.** $\frac{c}{a+1}$.

18. $\frac{x(1-a)}{2a}$. **19.** $\frac{x-y}{2}$. **20.** $\frac{x}{2(a^2-ab+b^2)}$. **21.** $\frac{a}{(a+1)^2}$.

22. $\frac{1-c}{a-2}$. **23.** $\frac{b-5}{c-3}$. **24.** $\frac{x+y}{b-2a}$. **25.** $\frac{a}{b+c}$. **26.** c^2-d^2.

27. $\frac{a^2+a+1}{a(1+a)^2}$. **28.** $\frac{2}{a-2}$. **29.** $\frac{2}{a(c+1)}$. **30.** $\frac{8}{2c(b-1)}$.

In examples 1–30, the product of the quotient by the divisor will give the original dividend.

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31. $\frac{1}{4}$. **32.** $\frac{2a^2(a^2-1)}{a^2-4}$, $8a^2$. **33.** $\frac{1}{(x+1)^2}$. **34.** $\frac{2a(2a-3x)}{(2a+3b)}$.

35. $\frac{1}{a^3-2a^2x+2ax^2-x^3}$. **36.** $\frac{3bx(x-2ay)}{2a}$. **37.** $\frac{x^2-25}{x-4}$. **38.** $\frac{x+1}{x}$.

Simple Equations containing Two Unknowns.—Page 168.

1. $x=3$, $y=2$.
2. $y=2$, $x=4$.
3. $y=2$, $x=3$.
4. $x=4$, $y=3$.
5. $x=2$, $y=3$.
6. $y=5$, $x=3$.
7. $y=2$, $x=6$.
8. $y=5$, $x=2$.
9. $x=2$, $y=6$.
10. $x=3$, $y=6$.
11. $x=5$, $y=4$.
12. $x=4$, $y=6$.
13. $x=7$, $y=5$.
14. $x=3$, $y=7$.
15. $y=2$, $x=5$.
16. $y=7$, $x=3$.
17. $x=5$. $y=3$.
18. $y=5$, $x=6$.
19. $x=4$, $y=3$.
20. $x=3$, $y=5$.
21. $y=4$, $x=-5$.
22. $y=7$, $x=2$.
23. $y=3$, $x=6$.
24. $x=4$, $y=5$.
25. $x=6$, $y=2$.
26. $y=9$, $x=8$.
27. $y=12$, $x=10$.
28. $y=10$, $x=12$.
29. $y=10$, $x=15$.
30. $y=12$, $x=12$.
31. $y=6$, $x=8$.
32. $y=8$, $x=16$.
33. $y=10$, $x=12$.
34. $y=5$, $x=9$.
35. $y=5$, $x=6$.
36. $y=20$, $x=30$.

Problems.—Page 170.

1. 8, 4. **2.** 4, 16. **3.** 20, 10. **4.** 8, 12. **5.** 35, 25.

6. Let $\frac{x}{y}=$ the fraction,
Then (1) $\frac{x-2}{y-2}=\frac{1}{3}$
And (2) $\frac{x+3}{y+3}=\frac{3}{4}$
Clearing of fractions (1) $3x-6=y-2$
(2) $4x+12=3y+9$
Transposing and collecting (3), $3x-y=4$
(4) $4x-3y=-3$
Multiplying (3) by 3 (5), $9x-3y=12$
Subtracting (4) from (5), $5x=15$
And $x=3$
Substituting the value of x in 1, $9-6=y-2$
Transposing and collecting, $-y=-5$
Or $y=5$
And $\frac{x}{y}=\frac{3}{5}$

7. $\frac{x}{y}=\frac{3}{10}$. **8.** $\frac{x}{y}=\frac{5}{8}$. **9.** 10 pigeons, Harry; 8 pigeons, Fred. **10.** 10 years, son; 35 years, father. **11.** 20 cents in one box; 30 cents in the other.

12. Let $x =$ Mary's number of pieces,
And $y =$ Martha's number of pieces.
Also (1) $x = y$
Then (2) $5x + y = 60$ cents
And (3) $\underline{5x - 5y = 0}$
Subtracting, $6y = 60$
And $y = 10$
Substituting the value of y in (1), $x = 10$
Therefore, Mary receives 50 cents, and Martha 10 cents.

13. $32, Anna paid; $48, Ellen paid. **14.** $1, George; $2, Henry. **15.** $5, each girl; $8, each boy. **16.** 2 cents, each apple; 3 cents, each orange. **17.** $1.20, wheat; $.60, corn. **18.** 5 two-dollar notes; 5 five-dollar notes. **19.** 50 cents, Emma; 70 cents, Ida. **20.** 60 sheep in first field; 20 sheep in second.

21. Let $x =$ tens' digit,
And $y =$ ones' digit.
Then (1) $x + y = 9$
And (2) $\underline{10x + y + 9 = 10y + x}$
Transposing and collecting
(2) (3), $9x - 9y = -9$
Multiplying (1) by 9 (4), $\underline{9x + 9y = 81}$
Subtracting (4) from (3), $-18y = -90$
Or $y = 5$
Substituting y in (1), $x + 5 = 9$
And $x = 4$
Hence, $10x + y = 45$
Therefore, the number is 45.

22. 72.

23. Let $x =$ the principal,
And $y =$ the rate,
Then (1) $x + \frac{4xy}{100} = \$600$
And (2) $\underline{x + \frac{8xy}{100} = \$700}$
Multiplying (1) by 2 (3), $2x + \frac{8xy}{100} = 1200$
Subtracting (2) from (3), $x = \$500$
Substituting x in (1), $500 + 20y = 600$

Transposing, $20y = 100$

And $y = 5$

Therefore, the principal is \$500, and the rate is 5%.

24. \$3000; \$2000.

Review Examples and Problems.—Page 172.

1. $27b^3+18b^2x-12bx^2-8x^3$. **2.** $8x^3-12x^2y-6xy^2+y^3$. **3.** $3x^4-x^2+x-1$. **4.** $1-2c^2+c^4$. **5.** $a^4+a^2x^2+x^4$. **6.** $x^4-6x^3+9x^2-16$. **7.** $a^4-9a^2b^2+24a^2b^3-16a^2b^4$. **8.** $9x^8-19x^4y^2+y^4$. **9.** $81a^4-288a^2y^2+256y^4$. **10.** x^6-y^6. **11.** a^6-y^6. **12.** $x^8+2x^6+x^4-1$. **13.** $1-x^6$.

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14. $x^2-[xy-y^2-xy+x^2]-y^2-2xy=-2xy$. **15.** $2a^2+2a-(2a-2)-2=2a^2$. **16.** $(1+2b+b^2-x^2)-2b+x^2-b^2=1$. **17.** $(2bc+2bd)-(2ac+2bd)=2c(b-a)$. **18.** x^2+y^2. **19.** $-x^2-x+1$. **20.** $x^2+2xy+y^2$. **21.** $1+2y+3y^2+2y^3+y^4$. **22.** $a+b-c$. **23.** x^2+x+1. **24.** $x-y-z+\frac{2yz}{x+y+z}$. **25.** $1+2a+3a^2+2a^3+a^4$. **26.** $1-2x+x^2$. **27.** $2x^3-3x^2+2x$. **28.** $a^4-a^3+a^2-a+1$. **29.** $2x+2y$. **30.** $a^2-2ab+2b^2$. **31.** $3x+y$. **32.** $a^3+a^2b+ab^2+b^3$. **33.** $x^4-x^3+x^2-x+1$. **34.** $8x^3+16x^2+32x+64$.

In problems 18–34, the products of the divisors by the quotients will be the original dividends.

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35. H. c. f. $=3x-2y$; $3x-2y$, $3x+2y$, $3x+4y$. **36.** H. c. f. $=b(2b+3)$; 1, $2(2b-3)$, $3b(2b-6)$. **37.** H. c. f. $=ab(2x-1)$; 1, $a(2x-1)$, $ab(4x^2+2x+1)$. **38.** H. c. f. $=2-3b$; $2x(3-3b)$, $x^2(4+6b+9b^2$, $(3-2b)$. **39.** L. c. m. $=12b^2x(4-x^2)(4-x)$; $3bx(2+x)(4-x)$, $6bx(4-x)$, $4(2+x)$; h. c. f. $=b(2-x)$. **40.** L. c. m. $=4abx^2(x^3-3x^2-9x+27)$; $x^2(x-3)(x+3)$, $2abx(x+3)$, $(x-3)$; h. c. f. $=2(x-3)$. **41.** L. c. m. $=5x^2(b^3-7b^2-25b+175)$; $5x^2(b-5)(b-7)$, $5x(b-7)$, $(b-5)$; h. c. f. $=b+5$. **42.** L. c. m. $=4a^2(x+y)^2(x-2y)$; $4a^2(x+y)(x-2y)$, $4a^2(x-2y)$, $(x+y)$; h. c. f. $=x+y$.

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43. $x+\frac{a}{x+y}$. **44.** $a-x$. **45.** $b-2y+\frac{3y^2}{b+y}$. **46.** $a+c+\frac{b^2}{a-c}$. **47.** $2x-6$. **48.** $3x+4y$.

In examples 43–48, the required operations give the original fractional form.

49. $\frac{2a^2 - 2ax + 2x^2}{a-x}$; $2a + \frac{2x^2}{a-x}$. **50.** $\frac{6x^2 + 4y^2}{3x + 2y}$; $2x - \frac{4xy - 4y^2}{3x + 2y}$.
51. $\frac{8a^2 + 18ax + 9x^2}{2a+3x}$; $4a + 3x$. **52.** $\frac{12x^2 + 2x - 6}{4x-3}$; $3x + \frac{11x-6}{4x-3}$.
53. $\frac{4a^2+6ax}{2a-3x}$; $2a + \frac{12ax}{2a-3x}$. **54.** $\frac{3x^2-7x-15}{2x+3}$; $1\tfrac{1}{2}x - 5\tfrac{3}{4} + \frac{2\frac{1}{4}}{2x+3}$, or
$-5 + x + \frac{x^2}{3+2x}$.

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55. $\frac{5x+5}{12}$; $\frac{5x-20+x^2}{12}$. **56.** $\frac{2a+1}{a^2-1}$; $\frac{a}{a^2-1}$. **57.** 1; $\frac{2x}{x+y}$.
58. $\frac{2x^2+8}{x^2-4}$; $\frac{2(x-2)}{x+2}$.

Changing required operations in addition and subtraction in 55–58:—

55. $\frac{x+25}{12}$; $\frac{x+50-x^2}{12}$. **56.** $\frac{-2a+5}{a^2-1}$; $\frac{-a+6}{a^2-1}$. **57.** $\frac{x^2+y^2}{x^2-y^2}$;
$\frac{2xy}{x^2-y^2}$. **58.** $\frac{8x}{x^2-4}$; $\frac{16x}{x^2-4}$.

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59. $\frac{1}{a}$. **60.** $\frac{x+2}{a(1-c)}$. **61.** $\frac{2}{x+3}$. **62.** $-\frac{x(x+2)}{x-2}$. **63.** $\frac{a^2+4a+4}{2a}$;
$\frac{a+2}{4}$. **64.** $\frac{x+2}{1}$; $x + 1 + \frac{4}{x^2}$. **65.** $\frac{a-1}{a+1}$; $-\frac{4a}{a^2-1}$.

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66. Clearing of fractions, $4x + 36 - 9x - 81 = 4x + 36 - 108$
$12 + 36 - 27 - 81 = 12 + 36 - 108$
Collecting, $-9x = -27$
And $x = 3$

In 66–70, substitute the value of x, and verify each equation.

67. $x = 10$. **68.** $x = 12$. **69.** $x = 6$. **70.** $x = 4$. **71.** $x = 3$.
72. $x = 2a$. **73.** $x = a + b + \frac{b^2}{a-b} = \frac{a^2}{a-b}$. **74.** $x = \frac{a^2}{b+c}$.
75. $x = b$. **76.** $x = 3(a - b)$.

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77. 32. **78.** 10, 12. **79.** 20 pounds @ 20 cents; 40 pounds @ 35 cents. **80.** 3 days.

81. Let x = sum at first,
Then $x-5$ = sum after spending,
And $2x-10$ = sum after borrowing,
Hence, $2x-15$ = sum after spending second time,
And $4x-30$ = sum after borrowing second time,
Also, $4x-35$ = sum after spending third time.

$$4x-35=5$$
$$x=10$$

Therefore, he had 10 dollars at first.

82. \$1000, B; \$800, A. **83.** \$60. **84.** $x=\frac{abn}{a+b}$; $a=3$, $b=9$, $n=8$, $x=18$.

85. Let x = time by C,

Then $\frac{1}{n}-\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{1}{x}$

And $\frac{1}{n}-\left(\frac{a+b}{ab}\right)=\frac{1}{x}$

Or $\frac{1}{n}-\frac{a+b}{ab}=\frac{1}{x}$

$$abx-anx-bnx=abn$$
$$x=\frac{abn}{ab-an-bn}$$

$n=1\frac{1}{3}$, $a=6$, $b=4$, $x=3$ days.

86. Let x = distance,

Then $\frac{x}{a+b}+\frac{x}{a-b}=c$

$$(a-b)x+(a+b)x=c(a^2-b^2)$$
$$x=\frac{c(a^2-b^2)}{2a}$$

$a=6$, $b=4$, $c=12$, $x=20$ miles.

87. $x=\frac{d-b}{a-c}$; $a=2$, $b=20$, $c=3$, $d=10$, $x=10$. **88.** $x=\frac{bn^2}{a+bn}$, at home; $n-x=\frac{an}{a+bn}$, number went; $n=4$, $a=12$, $b=1$, $x=1$, at home. **89.** $x=\frac{an-c}{a+b}$, days idle; $n-x=\frac{bn+c}{a+b}$ = days worked; $a=3$, $b=1$, $n=20$, $c=50$, $x=2\frac{1}{2}$. **90.** $x=b+c$, $y=b-c$. **91.** $y=1$, $x=5$. **92.** $x=\frac{a+b}{2}$, $y=\frac{a-b}{2}$. **93.** $x=\frac{c+d}{a}$, $y=\frac{c-d}{b}$. **94.** $x=\frac{2}{a+b}$, $y=\frac{2}{a-b}$.

In 95–100, substitute the values of x and y, and verify each equation.

95. $y=2$, $x=5$. **96.** $y=4$, $x=6$. **97.** $y=3$, $x=9$. **98.** $y=2$, $x=5$. **99.** $y=2$, $x=6$. **100.** $y=2$, $x=3$.

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101. 2 dozen @ 25 cents; 5 dozen @ 30 cents.

102. Let $x=$ tens' digit,
And $y=$ ones' digit,
Then $\frac{10x+y}{x+y}=4$
And $10x+y+27=10y+x$
$x=3$
$y=6$
$10x+y=36$

103. $y=90$ marbles, Harry; $x=70$ marbles, Frank. **104.** $x=\$100$, $y=5$ yr. **105.** $y=18$, Fred's; $x=30$, John's. **106.** $x=24$, at 3 for 5 cents; $y=24$, at 2 for 5 cents.

107. Let $x=$ rate on still water,
And $y=$ rate of current,
Then $2x+2y=15$ miles,
And $6x-6y=15$ miles,
$x=5$ miles,
$y=2\frac{1}{2}$ miles.

108. Let $x=$ number of men,
And $y=$ cost per man,
Then $(x+6)(y-1)=xy$,
And $(x-3)(y+1)=xy$.
$y=\$3$ per man,
$x=12$ men.

109. $y=5$ inches, width; $x=8$ inches, length. **110.** $x=\frac{an+am}{2mn}$; $y=\frac{an-am}{2mn}$, rate of current.

111. Let $x=$ the principal,
And $y=$ the rate,
Then (1) $x+\frac{xmy}{100}=a$,
And (2) $x+\frac{xny}{100}=b$,
Multiplying (1) (3), $nx+\frac{mnxy}{100}=an$,

By n and (2) by (4) $mx+\dfrac{mnxy}{100}=bm.$

Substituting $\quad (n-m)x=an-bm,$

And $\quad x=\dfrac{an-bm}{n-m},$

Hence, $\quad y=\dfrac{100(b-a)}{an-bm}.$

The difference in the amounts is the difference in interest for the difference in time; hence, the rate = [difference in amounts or interest] ÷ (prin. × difference in time).

112. $y=\dfrac{am-bn}{a^2-b^2}$, $x=\dfrac{bm-an}{b^2-a^2}$. **113.** $y=\dfrac{m(a-c)}{a-b}$, $x=\dfrac{m(b-c)}{b-a}$.

Powers and Roots.—Page 180.—Involution.

1. $25x^4y^2$. **2.** $64a^6x^9$. **3.** $625a^8y^8$. **4.** $-a^9b^6c^3$. **5.** $8a^3b^6c^9$. **6.** $1296x^8y^4z^{12}$. **7.** $16a^8c^4x^{12}z^8$. **8.** $243a^{10}x^{15}y^{10}z^{15}$. **9.** $64b^6c^{12}x^{18}y^{12}$. **10.** $\dfrac{16a^4c^8}{25c^6d^4}$. **11.** $-\dfrac{8d^9e^6}{216a^6b^9}$. **12.** $\dfrac{8a^6x^9y^6}{64x^6y^3z^9}$. **13.** $\dfrac{c^8o^{12}n^8}{m^{12}n^8s^{12}}$. **14.** $\dfrac{81b^{12}c^8}{16b^8x^4y^{12}z^8}$. **15.** $-\dfrac{x^{10}y^{15}}{a^{15}c^{10}x^{10}y^{15}}$.

To Raise a Polynomial to any Power.

1. a^4+2a^2+1. **2.** $x^3-6x^2y+12xy^2-8y^3$. **3.** $8a^3+36a^2b+54ab^2+27b^3$. **4.** $27x^3-108x^2y+144xy^2-64y^3$. **5.** $81x^4+108x^3y+54x^2y^2+12xy^3+y^4$. **6.** $16a^4b^8-96a^3b^6c+216a^2b^4c^2-216ab^2c^3+81c^4$. **7.** $\dfrac{x^2}{4}+2xy+4y^2$. **8.** $27a^6b^3-13\frac{1}{2}a^4b^2x+2\frac{1}{4}a^2bx^2-\dfrac{x^3}{8}$. **9.** $x^2+y^2+z^2-2xy-2xz+2yz$. **10.** $x^2+4y^2+9z^2+4xy-6xz-12yz$. **11.** $a^2+4b^2+c^2+4ab+2ac+4bc$. **12.** $27x^3+8y^3-z^3+54x^2y-27x^2z+36xy^2-36xyz+9xz^2-12y^2z+6yz^2$.

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1. $a^2+2ax+x^2$. **2.** $x^2-2xy+y^2$. **3.** $4a^2+4ab+b^2$. **4.** $a^2-4ab+4b^2$. **5.** $a^3+3a^2x+3ax^2+x^3$. **6.** $a^3-3a^2y+3ay^2-y^3$. **7.** $8a^3+12a^2b+6ab^2+b^3$. **8.** $a^3-6a^2b+12ab^2-8b^3$. **9.** $x^4+4x^3y+6x^2y^2+4xy^3+y^4$. **10.** $x^4-4x^3y+6x^2y^2-4xy^3+y^4$. **11.** $a^4+8a^3x+24a^2x^2+32ax^3+16x^4$.

12. $16a^4-32a^3x+24a^2x^2-8ax^3+x^4$. **13.** $a^5+5a^4x+10a^3x^2+10a^2x^3+5ax^4+x^5$. **14.** $x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-y^5$. **15.** $b^5+5b^4y+10b^3y^2+10b^2y^3+5by^4+y^5$. **16.** $32a^5+80a^4b+80a^3b^2+40a^2b^3+10ab^4+b^5$.

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1. 169. **2.** 225. **3.** 484. **4.** 1225. **5.** 9261. **6.** 15625. **7.** 50625. **8.** 160000. **9.** 248832. **10.** 100000. **11.** 91125. **12.** 1953125.

To find the Square of any Polynomial.

1. $b^2+c^2+d^2+2bc+2bd+2cd$. **2.** $d^2+e^2+f^2+2de-2df-2ef$. **3.** $x^2+y^2+z^2-2xy-2xz+2yz$. **4.** $r^2+s^2+4t^2+2rs+4rt+4st$. **5.** $x^2+4y^2+9z^2+4xy-6xz-12yz$. **6.** $a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc+2bd+2cd$.

Evolution.—Page 183.—To find any Root of a Monomial.

1. $6x^2y^3$. **2.** $5ax^4$. **3.** $5a^2b$. **4.** $6rs^2$. **5.** $-7ab^3$. **6.** $3b^2c^3$. **7.** $4x^2yz^3$. **8.** $4a^2b^3$. **9.** $6x^3y^3$. **10.** $\frac{4x^2y}{5az^3}$. **11.** $\frac{2ax^2}{3x^2y}$. **12.** $-\frac{4a^2b^3}{5b^2c}$. **13.** $\frac{2ab^2}{3x^2y^3}$. **14.** $-\frac{5ab^3c^2}{6b^2xy^3}$. **15.** $\frac{3ab^2c^3}{4m^3no^2}$.

To find the Square Root of a Polynomial.

1. $2a+b$. **2.** $2b-c$. **3.** $c-2y$. **4.** $3a+b$. **5.** $3x-4y$. **6.** $4a-5b$. **7.** x^2+x+1. **8.** $1+a+2a^2$. **9.** $3x^3-2x^2+x$. **10.** $5x^2y-3xy^2+2y^3$. **11.** $a^2b^2-2ab+1$. **12.** $3x^3+4xy-3y^3$.

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1. 26. **2.** 29. **3.** 31. **4.** 33. **5.** 25. **6.** 55. **7.** 51. **8.** 74. **9.** 65. **10.** 81. **11.** 101. **12.** 153.

To find the Cube Root of a Polynomial.

1. $b-2c$. **2.** $2a+1$. **3.** $3b-2$. **4.** $3x+4y$. **5.** $4x-3y$. **6.** $5a-3$. **7.** x^2-2x+1. **8.** y^2+2y+3. **9.** $1-2x-x^2$. **10.** $2a^2-a+2$. **11.** $2x^2-3x-4$. **12.** $3x^2y^2+2xy-1$.

1. 6. **2.** 7. **3.** 8. **4.** 9. **5.** 12. **6.** 13. **7.** 25. **8.** 33. **9.** 35. **10.** 65.

Quadratic Equations.—Page 189.—To solve a Pure Quadratic.

In examples 1–10, substitute the values of x, and verify each equation.

1. $x=\pm 4$. **2.** $x=\pm 3$. **3.** $x=\pm 4$. **4.** $x=\pm 5$. **5.** $x=\pm 3$. **6.** $x=\pm 4$. **7.** $x=\pm 3$. **8.** $x=\pm 2$. **9.** $x=\pm 4$. **10.** $x=\pm 6$. **11.** $x=\pm 4$. **12.** $x=\pm \frac{1}{4}$. **13.** $x=\pm 3$. **14.** $x=\pm 10$. **15.** $x=\pm 2$. **16.** ± 20. **17.** ± 6. **18.** 20 in., length; 15 in., width. **19.** 20 pounds; 30 cents, price per pound. **20.** 25 cows, $10.

To solve an Affected Quadratic.

In examples 1–16, substitute the values of x, and verify each.

1. $x^2+2x=15$

Completing square, $x^2+2x+1=16$

Extracting square root, $x+1=\pm 4$

And $x=3 \text{ or } -5$

Substituting value of x, $\begin{cases} 9+6+1=16 \\ 25-10+1=16 \end{cases}$

2. 6 or -2. **3.** -2 or -3. **4.** 5 or -3. **5.** 2 or -4. **6.** 2 or -6. **7.** 6 or 4. **8.** 3 or -6. **9.** 1 or -9. **10.** 2 or -4. **11.** 1 or -2. **12.** -2 or -4. **13.** 5 or -3. **14.** -1 or -2. **15.** 3 or -1. **16.** 4 or $-1\frac{1}{2}$. **17.** $\frac{1}{2}$ or $-\frac{1}{4}$. **18.** $\frac{1}{4}$ or $-\frac{1}{2}$. **19.** $\frac{1}{2}$ or $-\frac{1}{3}$. **20.** $\frac{1}{3}$ or $-\frac{2}{3}$. **21.** 3 or $\frac{1}{3}$. **22.** 20 or $+1$. **23.** 7 or -1. **24.** $3\frac{2}{5}$ or 2. **25.** $\frac{1}{3}$ or $-1\frac{2}{3}$. **26.** 3 or -6.

27. 5 or $-5\frac{1}{2}$; 15 or $-16\frac{1}{2}$. **28.** $x=2$ or -6; $x+4=6$ or -2. **29.** 30 cents; 20 cents. **30.** 18 or -15, larger. **31.** 5 desks in a row; 8 rows. **32.** 20 rd.; 32 rd. **33.** 3 miles per hour. **34.** 12 days, B; 4 days, A. **35.** 18 ft. length, 16 ft. breadth; 12 feet. **36.** 5 ft., base; 12 ft., perpendicular; 13 ft., hypothenuse. **37.** $20. **38.** 6 boxes. **39.** 13 in., length; 9 in., breadth. **40.** $\frac{2}{3}$.

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